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RESEARCH ARTICLE

Bequests, estate taxes, and wealth distributions

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Abstract Bossmann et al. (J Public Econ 91:1247–1271, 2007) found that estate taxes reduce the long-run wealth inequality. This result contrasts with the findings of the previous literature with idiosyncratic labor efficiency risk. We use a decomposition technique, developed by Davies (J Labor Econ 4:538–559, 1986), to reinvestigate the impact of estate taxes on the long-run wealth inequality. We find that the different results of estate taxes are due to the different redistribution effects.

Keywords Wealth inequality · Bequest motives · Estate taxes · Lorenz dominance

JEL Classification D31 · E21 · H23

1 Introduction

Bossmann et al. (2007) found that estate taxes reduce the long-run wealth inequality. This result contrasts with the findings of the previous literature with idiosyncratic labor efficiency risk, such as Becker and Tomes (1979) and Davies (1986). These papers

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show that estate taxes usually increase the long-run wealth inequality.[1] In this paper we use the decomposition technique developed by Davies (1986) to reinvestigate the impact of estate taxes on the long-run wealth inequality. We find that the redistribution effect plays an important role in determining the effect of the estate tax on the long-run wealth inequality.

Following Bossmann et al. (2007), we investigate the impact of bequest motives and estate taxes on wealth inequality in a two-period overlapping generations (OLG) heterogeneous agents model. Each agent lives for two periods: the young period and the old period. There are a continuum of measure 1 families in the economy. Each family consists of one parent and one child. Each young agent supplies 1 unit of labor inelastically and has idiosyncratic labor efficiency risk. Old agents do not have labor earnings. Agents have "joy of giving" bequest motives. The government collects the estate tax revenue and gives a lump-sum transfer to the young generation. The government has a balanced budget in every period.

Becker and Tomes (1979) found that a progressive tax subsidy system tends to increase the long-run inequality. Davies (1986) developed a decomposition technique to study the impact of the estate tax on the long-run wealth inequality in the Becker–Tomes model. He separates the inheritance effect of the estate tax from the redistribution effect. And both of these two effects increase the long-run wealth inequality. Thus, estate taxes increase the long-run wealth inequality in the Becker–Tomes model.

As Bossmann et al. (2007), we find that estate taxes reduce the long-run wealth inequality. We use the decomposition technique, developed by Davies (1986), to reinvestigate the impact of estate taxes on the long-run wealth inequality. In our model the inheritance of bequests decreases the long-run wealth inequality through averaging labor efficiency luck in a lineage. The inheritance effect of the estate tax increases the long-run inequality through interfering with the inheritance of bequests. In this respect, our model and Bossmann et al. (2007) are in line with the previous literature with idiosyncratic labor efficiency risk, such as Becker and Tomes (1979) and Davies (1986).

We find that it is the redistribution effect of the estate tax that causes different effects on the long-run wealth inequality. The redistribution effect is the effect of the lump-sum transfer on wealth inequality. The increase in the transfer reduces wealth inequality. Agents in our model have "joy of giving" bequest motives. Raising estate taxes increases government revenues and subsidies. The redistribution effect decreases wealth inequality in our model and Bossmann et al. (2007).

In our model the inheritance effect of the estate tax and the redistribution effect work in opposite directions. The redistribution effect dominates the inheritance effect. The net effect of the estate tax is to reduce wealth inequality. These findings help us to understand the impact of estate taxes on the long-run wealth inequality. Davies (1986) found that the inheritance effect of the estate tax increases the long-run wealth inequality in the model with idiosyncratic labor efficiency risk. This result holds for both altruistic bequest motives and "joy of giving" bequest motives, as long as the wealth

[1] Simulations of Davies and Kuhn (1991) show that estate taxes reduce wealth inequality in the short run, even though they increase inequality in the long run.

accumulation equation is linear. Thus, the redistribution effect plays an important role in determining the net effect of the estate tax on wealth inequality. If the redistribution effect increases wealth inequality, as in Becker and Tomes (1979) and Davies (1986), then the net effect of the estate tax increases the long-run wealth inequality. If the redistribution effect decreases wealth inequality and it dominates the inheritance effect, as in our model and Bossmann et al. (2007), then the net effect of the estate tax reduces the long-run wealth inequality.

We also extend our benchmark model in two directions. In the first extension, we include housing as a new asset in the model. In this extension we show that all the theoretical results of the long-run wealth inequality in the benchmark model still hold. In the other extension, we permit the agent to live for more than two periods. In this extension we use a calibration exercise to illustrate that the results of the long-run wealth inequality in our benchmark model are still true. Estate taxes reduce the long-run wealth inequality.

Our findings also help us to understand how different ways of modeling bequest motives influence the impact of estate taxes on wealth inequality. Different forms of bequest motives, altruism and "joy of giving," do not influence the inheritance effect of the estate tax, but they imply different redistribution effects of the estate tax. In a model with "joy of giving" bequest motives the government can collect more tax revenues when it increases the estate tax. However, in a model with altruistic bequest motives the government collects less tax revenues when it increases the estate tax. Since a higher lump-sum transfer always reduces the wealth inequality, government revenues influence the long-run wealth distribution. Thus, different forms of bequest motives influence the impact of estate taxes on wealth inequality through the redistribution effect.

Although previous studies, such as Gale and Perozek (2001), find that different forms of bequest motives influence the impact of estate taxes on wealth accumulation, few papers investigate how different forms of bequest motives influence the impact of estate taxes on wealth inequality. Our paper fills this gap. This research is important given that empirical researches have not found evidences to distinguish these two bequest motives: altruism and "joy of giving." In a recent literature review, Kopczuk (2013) stated that "Bequest motives are the key building block for theoretical analysis of taxation of transfers, but the empirical literature has not settled on a clear answer to the question about the nature of bequest motivations" [page 331 of Kopczuk (2013)]. Pestieau and Thilbault (2012) investigated the long-run wealth distribution in an economy of agents with heterogeneous bequest motives.[2]

We do not incorporate precautionary savings motives into our model, and agents have linear policy functions.[3] The linear property permits us to use Lorenz dominance

[2] Mino and Nakamoto (2016) studied wealth inequality in an economy of consumption externalities and heterogeneous preferences.

[3] Studies of incomplete-market heterogeneous agents models, such as Aiyagari (1994), Castaneda et al. (2003) and De Nardi (2004), Benhabib et al. (2015), and De Nardi and Yang (2016), incorporate precautionary savings motives into their models. They solve agent's policy functions numerically and simulate the stationary wealth distribution. Benhabib et al. (2011) also found agent's policy functions explicitly. They use idiosyncratic investment risk to generate the observed fat tail of the wealth distribution in the USA.

to investigate the impact of bequest motives and estate taxes on wealth inequality. Lorenz dominance is widely used in the literature of income and wealth inequality. For example, Chatterjee (1994) used Lorenz dominance to discuss wealth distribution in a neoclassical growth model. Zilcha (2003) used Lorenz dominance to study the income distribution in an economy with two types of intergenerational transfers: investment of parents in the education of their offspring and capital transfer. Early studies include, among others, Atkinson (1970) and Rothschild and Stiglitz (1973). For a recent review on this topic see Gajdos and Weymark (2012).[4]

The rest of this paper is organized as follows. Section 2 presents the basic structure of our model. Section 3 discusses the stationary wealth distribution. In Sect. 4 we introduce housing into our benchmark model. We extend the benchmark model to a life cycle model in Sect. 5. Section 6 concludes the paper. Appendix contains most of the proofs.

2 The model

Our model is an overlapping generations heterogeneous agents economy. There are a continuum of measure 1 families in the economy. Each family consists of one parent and one child. Each agent lives for two periods: the young period and the old period. Each old agent gives birth to one child. The population of the economy keeps constant.[5]

2.1 The agent's problem

Young agents work and earn labor earnings. Each young agent supplies 1 unit of labor inelastically. But young agents have idiosyncratic labor efficiency risk l_t. We assume

Assumption 1 $\{l_t\}$ is stationary and ergodic.[6]

Assumption 2 $l_t > 0$ has a finite mean.[7] Without loss of generality,

$$E(l_t) = 1.$$

The wage rate per efficiency unit is w_t. An young agent born at period t consumes c_t^y in the first period of his life. s_t denotes his savings. The interest rate in period $t+1$ is r_{t+1}. An old agent does not have labor income. His consumption is c_{t+1}^o. He leaves the bequest b_{t+1} to his child in the second period of his life. The government collects the estate tax ζb_{t+1}, where $\zeta \in [0, 1)$ is the estate tax rate. In period t, an young

[4] Since our model has linear policy functions, wealth distribution does not influence the aggregate economy. Algan et al. (2011) built a model in which wealth redistribution can influence the aggregate output. Antunes et al. (2015) investigated the feedback of wealth distribution on the aggregate economy.

[5] Our model has a simple demographic structure. Modeling a more complicated demographic structure Mierau and Turnovsky (2014) studied the relationship between demography and wealth inequality.

[6] We use $\{x_t\}$ to represent a sequence in this paper.

[7] Note that we do not need to assume that $\mathrm{Var}(l_t) < \infty$.

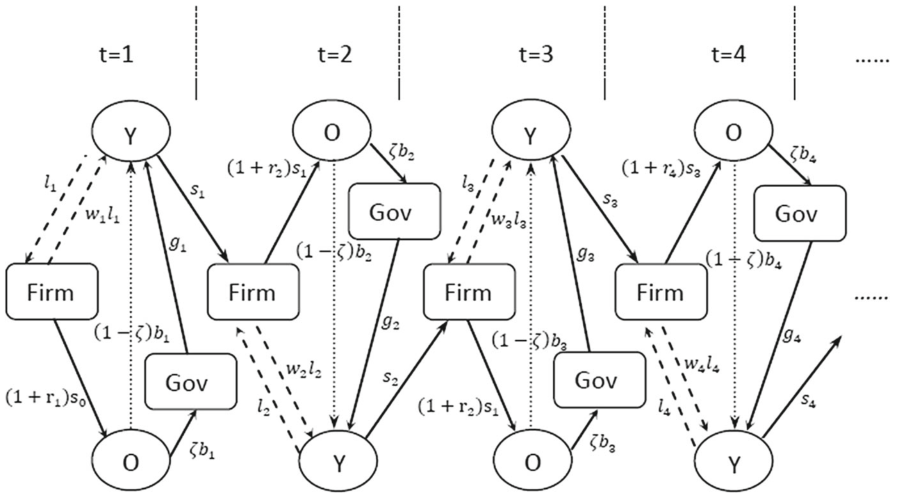

Fig. 1 The timing of the model

agent receives bequests $(1-\zeta)\,b_t$. He draws his labor efficiency l_t and receives the lump-sum transfer g_t from the government.

Figure 1 shows the timing of the model.

The young agent first draws his labor efficiency l_t, and then, the agent makes consumption and savings decisions. Thus, the agent's problem is a deterministic optimization problem. The old agent has a "joy of giving" bequest motive. Both the utility functions of the consumption and the bequest have the form of constant relative risk aversion (CRRA). The young agent's problem is

$$\begin{aligned}
&\max_{c_t^y, s_t, c_{t+1}^o, b_{t+1}} \frac{(c_t^y)^{1-\eta}-1}{1-\eta} + \beta\left[\frac{(c_{t+1}^o)^{1-\eta}-1}{1-\eta} + \chi\frac{\left[(1-\zeta)\,b_{t+1}\right]^{1-\eta}-1}{1-\eta}\right] \\
&\text{s.t. } c_t^y + s_t = w_t l_t + (1-\zeta)\,b_t + g_t, \\
&c_{t+1}^o + b_{t+1} = (1+r_{t+1})s_t, \qquad (1)
\end{aligned}$$

where $\eta \geq 1$ is the coefficient of relative risk aversion, $\beta \in (0, 1)$ is the time discount factor, and $\chi > 0$ represents the bequest motive intensity.

The agent's optimal policy functions are

$$\begin{aligned}
c_{t+1}^o &= \frac{1}{1+\chi^{\frac{1}{\eta}}(1-\zeta)^{\frac{1-\eta}{\eta}}}(1+r_{t+1})s_t, \\
b_{t+1} &= \frac{1}{1+\chi^{-\frac{1}{\eta}}(1-\zeta)^{\frac{\eta-1}{\eta}}}(1+r_{t+1})s_t, \\
c_t^y &= \frac{1}{1+\tilde{\beta}_{t+1}^{\frac{1}{\eta}}}\left[w_t l_t + (1-\zeta)\,b_t + g_t\right],
\end{aligned}$$

and

$$s_t = \frac{1}{1+\tilde{\beta}_{t+1}^{-\frac{1}{\eta}}} \left[w_t l_t + (1-\zeta)\, b_t + g_t \right],$$

where $\tilde{\beta}_{t+1} = \beta \left[1 + \chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}} \right]^{\eta} (1+r_{t+1})^{1-\eta}$.

From optimal policy functions of b_{t+1} and s_t, we derive the agent's wealth accumulation equation,

$$s_t = \frac{1}{1+\tilde{\beta}_{t+1}^{-\frac{1}{\eta}}} \left[w_t l_t + (1-\zeta)\, \varphi(1+r_t) s_{t-1} + g_t \right], \tag{2}$$

where $\varphi = \frac{1}{1+\chi^{-\frac{1}{\eta}} (1-\zeta)^{\frac{\eta-1}{\eta}}}$.

These linear functions, induced by the CRRA utility functions, bring us conveniences to describe both the aggregate economy and the stationary wealth distribution. Under the linear wealth accumulation equation, the aggregate economy only depends on the mean of the wealth accumulation. Other moments of the wealth distribution do not influence the aggregate economy. Thus, we can use a nonlinear equation to describe the aggregate capital accumulation without characterizing the wealth distribution along the transition of the aggregate economy.

When we investigate the stationary wealth distribution, the linear wealth accumulation equation permits us to find an expression of the stationary wealth distribution. We also use the linear wealth accumulation equation to establish the Lorenz dominance relationship when we study the comparative statics of the stationary wealth distribution.

2.2 The firm's problem

A firm has the aggregate production in the economy,

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha},$$

where A is the technology level, Y_t is the output, K_t is the capital, L_t is the labor, and α is capital's share of income. The firm chooses K_t and L_t to maximize its profits,

$$\max_{K_t, L_t} \{ A K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - (r_t + \delta) K_t \},$$

where δ is the depreciation rate of capital.

The capital market and the labor market are competitive. Capital and labor are paid their marginal products. From the firm's problem we have

$$r_t = \alpha A K_t^{\alpha-1} L_t^{1-\alpha} - \delta, \tag{3}$$

and

$$w_t = (1 - \alpha) A K_t^{\alpha} L_t^{-\alpha}. \tag{4}$$

2.3 The government

The government collects the estate tax revenue and gives a lump-sum transfer to the young generation. Each young agent receives the same subsidy g_t. The government has a balanced budget in every period. Thus, we have

$$g_t = \zeta \int b_t di, \tag{5}$$

where $\int di$ denotes the aggregation of young agents.

2.4 The general equilibrium

The aggregate population of young agents, who are the workers in the economy, is 1. And we have $E(l_t) = 1$ from Assumption 2. Thus, the labor market clearing condition is

$$L_t = \int l_t di = 1, \tag{6}$$

where $\int di$ denotes the aggregation of young population.

The capital market clearing condition is

$$K_{t+1} = \int s_t di, \tag{7}$$

where $\int di$ denotes the aggregation of young agents.

Aggregating Eq. (2) across young agents and using Eqs. (6) and (7), and the government budget constraint (5), we have

$$K_{t+1} = \frac{1}{1 + \tilde{\beta}_{t+1}^{-\frac{1}{\eta}}} \left[w_t + \varphi(1 + r_t) K_t \right]. \tag{8}$$

With the labor market clearing condition (6), Eqs. (3) and (4) imply that

$$r_t = \alpha A K_t^{\alpha - 1} - \delta, \tag{9}$$

and

$$w_t = (1 - \alpha) A K_t^{\alpha}. \tag{10}$$

Plugging Eqs. (9) and (10) we have

$$K_{t+1} = \frac{1}{1 + \tilde{\beta}_{t+1}^{-\frac{1}{\eta}}} \left[(1 - \alpha + \varphi\alpha) A K_t^{\alpha} + \varphi(1 - \delta) K_t \right]. \tag{11}$$

This nonlinear equation describes the law of motion of the aggregate capital.

We will concentrate on the steady-state aggregate economy in which the aggregate capital K, the wage rate w, and the interest rate r are constant.

Proposition 1 *The economy has a unique aggregate steady state. An economy with a higher bequest motive* χ *has a higher steady-state aggregate capital* K*.*

The young agent born in period t has two incentives for accumulating wealth. The first incentive is for his own consumption in the old period c^o_{t+1}. The second incentive is for the bequest left to his child b_{t+1} . The higher the agent's bequest motive, the higher the agent's saving incentive for the bequest. Thus, the wealth accumulation is higher. In one extreme case there is no bequest motive, i.e., $\chi = 0$. Proposition 1 implies that the steady-state aggregate economy with the bequest motive $\chi > 0$ has a higher aggregate wealth level than the economy without bequest motives.

3 The wealth distribution

In this section we investigate the stationary distribution of individual wealth accumulation process in the steady-state aggregate economy. Following Bossmann et al. (2007) we use a_{t+1} to denote the individual wealth (before being paid interest) in period $t+1$. Thus, $a_{t+1} = s_t$.

From government's budget constraint (5), we have

$$g_t = g = \zeta\varphi(1+r)K, \tag{12}$$

in the steady-state aggregate economy.

Plugging Eq. (12) into (2) we have the agent's wealth accumulation equation in the steady-state aggregate economy,

$$a_{t+1} = c_3 l_t + c_4 a_t + c_5, \tag{13}$$

where $c_3 = \frac{1}{1+\tilde{\beta}^{-\frac{1}{\eta}}} w$, $c_4 = \frac{(1-\zeta)\varphi(1+r)}{1+\tilde{\beta}^{-\frac{1}{\eta}}}$, and $c_5 = \frac{\zeta\varphi(1+r)}{1+\tilde{\beta}^{-\frac{1}{\eta}}} K$.

Equation (13) is the main equation of our paper. Our aim is to investigate the stationary distribution of process $\{a_t\}$ in the steady-state aggregate economy. We will study the stationary distribution of $\{a_t\}$ using the linear relationship of Eq. (13). We also use this linear relationship to compare the stationary wealth distribution of different economies in Sects. 3.1 and 3.2.

To study the stationary distribution of process $\{a_t\}$ we first characterize the coefficient c_4 in the wealth accumulation Eq. (13).

Proposition 2 $0 \leq c_4 < 1$.

Since Proposition 1 shows that aggregate capital K in the steady-state aggregate economy is finite, we have an intuitive way to understand Proposition 2. And the aggregate savings equal K. Suppose that $c_4 \geq 1$, then $a_t \to \infty$ almost surely as $t \to \infty$. Then, we have $K = \infty$. Thus, we must have $c_4 < 1$ in the steady-state aggregate economy.

Proposition 3 *The unique stationary distribution of* $\{a_t\}$ *is*[8]

$$a_\infty =_{st} c_3 \sum_{s=0}^{\infty} c_4^s l_s + \frac{c_5}{1-c_4}. \tag{14}$$

And a_t *converges to* a_∞ *in distribution, denoted by* $a_t \to_{st} a_\infty$*, as* t *approaches infinity.*

Proposition 3 shows that a_t converges to a_∞ in distribution as t approaches infinity. We use this important property of convergence in distribution when we investigate the impacts of bequest motives and estate taxes on the stationary wealth distribution. In these analyses we first establish intuitions and obtain results in static situations. Then, we extend these results to stationary wealth distributions by showing that they still hold when processes approach limiting distributions.

Bossmann et al. (2007) assumed that $\mathrm{Var}(l_t) < \infty$. However, as noted by Bossmann et al. (2007), "the assumption of a finite variance is not satisfied for all commonly used distributional assumptions for l_t" [Footnote 9 on page 1255 of Bossmann et al. (2007)]. To derive Proposition 3 we do not assume that $\mathrm{Var}(l_t) < \infty$.

3.1 Bequest motives and wealth inequality

In order to emphasize the impacts of bequest motives on wealth distribution, we set estate tax rate $\zeta = 0$, following Bossmann et al. (2007). Thus, $c_5 = 0$. The agent's wealth accumulation Eq. (13) becomes

$$a_{t+1} = c_3 l_t + c_4 a_t.$$

Following Bossmann et al. (2007), we assume that there are two economies: economy A and economy B. Agents in economy A do not have bequest motives, i.e., $\chi = 0$. Agents in economy B have bequest motives, i.e., $\chi > 0$ (B for bequests). Let a_∞^A be the stationary wealth distribution of economy A, and a_∞^B be the stationary wealth distribution of economy B.

In economy A there is no bequest motive and $c_4 = 0$. Thus, we have

$$a_{t+1} = c_3 l_t,$$

which has the same Lorenz curve as l_t. Thus, a_∞^A has the same Lorenz curve as l_t.

[8] Here $=_{st}$ denotes equality in distribution.

In economy B there are bequest motives. By Proposition 2 we have $0 < c_4 < 1$. Plugging $c_5 = 0$ into Eq. (14) we have

$$\begin{aligned} a_\infty^B &= {}_{st} c_3 \sum_{s=0}^{\infty} c_4^s l_s \\ &= {}_{st} \frac{c_3}{1-c_4} \sum_{s=0}^{\infty} (1-c_4) c_4^s l_s. \end{aligned}$$

Since $\frac{c_3}{1-c_4}$ is a constant, a_∞^B has the same Lorenz curve as $Z \equiv \sum_{s=0}^{+\infty}(1-c_4)c_4^s l_s$. The random variable Z is a weighted average of random variables, $l_0, l_1, l_2, \ldots$. It should be more equal than l_t.[9] Our analysis of the impacts of bequests on wealth distribution starts from this observation. We extend this intuition to the comparison between stationary wealth distributions a_∞^A and a_∞^B.

Let $L_X(p)$ be the Lorenz curve of a nonnegative random variable X with a finite positive mean.[10] Using the Lorenz curve, we define the Lorenz ordering.[11]

Definition 1 For two random variables X and Y, X Lorenz dominates Y if and only if $L_X(p) \geq L_Y(p)$, $p \in [0, 1]$, denoted as $X \succeq_L Y$.

Another commonly used inequality measure is the coefficient of variation (CV). For a random variable X with a finite variance, CV(X) is defined as

$$\mathrm{CV}(X) = \frac{\sqrt{\mathrm{Var}(X)}}{E(X)}.$$

If both X and Y have finite variances, then $X \succeq_L Y$ implies $\mathrm{CV}(Y) \geq \mathrm{CV}(X)$.[12] Thus, Lorenz dominance implies the order of the coefficient of variation.

Comparing stationary wealth distributions a_∞^A and a_∞^B, we have

Theorem 1 *Under Assumptions 1 and 2,* $a_\infty^B \succeq_L a_\infty^A$.

Theorem 1 implies that, an economy in which agents have bequest motives has a more equal wealth distribution than an economy in which agents do not have bequest motives. Theorem 1 is the same as Proposition 1 of Davies (1986). While Davies (1986) used altruistic bequest motives as in Becker and Tomes (1979), our paper uses "joy of giving" bequest motives as in Bossmann et al. (2007).

Bossmann et al. (2007) also found that an economy with bequest motives has a stationary wealth distribution which is more equal than that of an economy without bequest motives. Our result extends that of Bossmann et al. (2007) in three respects. First, we only assume that $\{l_t\}$ is a stationary and ergodic. Bossmann et al. (2007)

9 We state this intuition formally in Lemma 2 of "Appendix A.4".

10 For the mathematical definition of the Lorenz curve, $L_X(p)$, see Gastwirth (1971).

11 For inequality measures, our main reference book is Shaked and Shanthikumar (2010). A good reference of Lorenz dominance is Arnold (1987).

12 See pages 68–69 of Marshall and Olkin (2007).

assume that $\{l_t\}$ either is independent and identically distributed ($i.i.d.$) or follows a linear mean-reverting process.[13] Secondly, we do not assume that $\mathrm{Var}(l_t) < \infty$. Bossmann et al. (2007) used the coefficient of variation as their inequality measure. Thus, they need $\mathrm{Var}(l_t) < \infty$ to insure that the variance of the wealth distribution is finite. Our inequality measures are the Lorenz curve and the Gini coefficient, which only require the existence of the mean of the wealth distribution. Thus, we do not need $\mathrm{Var}(l_t) < \infty$. Finally, our result implies that of Bossmann et al. (2007). Bossmann et al. (2007) derived the coefficient of variation of wealth, their inequality measure, by calculating the mean and variance of the wealth distribution. They find that $\mathrm{CV}(a_\infty^A) > \mathrm{CV}(a_\infty^B)$ because the increase in mean wealth "overcompensates the increase in the variance of the wealth" [Page 1249 of Bossmann et al. (2007)]. Thus, the coefficient of variation falls.[14] If $\mathrm{Var}(a_\infty^A) < \infty$ and $\mathrm{Var}(a_\infty^B) < \infty$, our Theorem 1 implies that $\mathrm{CV}(a_\infty^A) \geq \mathrm{CV}(a_\infty^B)$.[15]

3.2 Estate taxes and wealth inequality

When investigating the impacts of estate taxes on wealth distribution, we concentrate on the logarithmic utility as in Bossmann et al. (2007).

Assumption 3 Utility functions are logarithmic.

Let $\eta = 1$. Then, the CRRA utility of Sect. 2 reduces to the logarithmic utility. We can solve the aggregate capital of the steady-state aggregate economy,

$$\bar{K} = \left(\frac{1-\alpha+\chi}{1+\frac{1}{\beta}+\delta\chi} A \right)^{\frac{1}{1-\alpha}} .$$

The estate tax does not affect aggregate capital. Thus, it does not influence the interest rate and the wage rate since we have $\bar{r} = \alpha A \left(\bar{K}\right)^{\alpha-1} - \delta$ and $\bar{w} = (1-\alpha)A\left(\bar{K}\right)^{\alpha}$ in the equilibrium. The estate tax has no general equilibrium effect.

From Eq. (13) we know that the individual wealth accumulation equation is

$$a_{t+1} = c_6 l_t + c_7 \left[(1-\zeta)a_t + \zeta \bar{K}\right], \tag{15}$$

with $c_6 = \frac{1}{1+\frac{1}{\beta(1+\chi)}}\bar{w}$ and $c_7 = \frac{1}{\left(1+\frac{1}{\beta(1+\chi)}\right)\left(1+\frac{1}{\chi}\right)}(1+\bar{r})$. Both c_6 and c_7 do not depend on the estate tax ζ.

[13] The linear process is

$$l_{t+1} = \bar{l} + v(l_t - \bar{l}) + \varepsilon_{t+1}$$

where $\bar{l} = 1$ and $0 < v < 1$. $\{\varepsilon_t\}$ is $i.i.d.$ with a zero mean, a finite variance, and a lower bound sufficient to keep $l_{t+1} > 0$. This process is used in Davies (1986) and Davies and Kuhn (1991). Solon (1992) and Zimmerman (1992) used different datasets in the USA to study the intergenerational mobility and found that the elasticity of child's earnings with respect to parent's earnings is about 0.4.

[14] See comments after Proposition 1 about the increase in mean wealth caused by bequest motives.

[15] There is one minor difference between our result and that of Bossmann et al. (2007). We can only show $\mathrm{CV}(a_\infty^A) \geq \mathrm{CV}(a_\infty^B)$, while they show $\mathrm{CV}(a_\infty^A) > \mathrm{CV}(a_\infty^B)$.

Table 1 The inheritance effect of the estate tax

Models	Bequest motives	The inheritance effect of the estate tax
This paper Bossmann et al. (2007)	"joy of giving"	To increase the long-run wealth inequality
Becker and Tomes (1979) Davies (1986)	Altruism	To increase the long-run wealth inequality

We use Proposition 3 to represent the stationary wealth distribution as

$$a_{\infty} = {}_{st} \sum_{s=0}^{\infty} c_8^s \left(c_6 l_s + c_7 \zeta \bar{K} \right), \tag{16}$$

where $c_8 = c_7(1 - \zeta)$. Davies (1986) used a decomposition technique to investigate channels through which the estate tax influence the long-run wealth inequality in Becker and Tomes (1979). I use this decomposition technique to analyze these two channels in our model. The channel through which the estate tax influences c_8 is called the lag structure effect. The channel through which the estate tax influences the term $c_6 l_s + c_7 \zeta \bar{K}$ is called the transfer effect. Thus, we separate the inheritance effect of the estate tax from the redistribution effect. The lag structure effect represents the inheritance effect. Also the government collects the estate tax revenue and then gives a lump-sum transfer to the young generation. Thus, the transfer effect represents the redistribution effect.

3.2.1 The inheritance effect of the estate tax

Theorem 1 shows that the inheritance of bequests reduces the long-run wealth inequality through intergenerational sharing of labor efficiency luck in a lineage. The higher the estate tax ζ, the smaller c_8 of Eq. (16). The wealth distribution becomes less equal due to the inheritance effect of the estate tax.

Davies (1986) uses altruistic bequest motives to study the inheritance effect of the estate tax on the long-run wealth inequality in Becker and Tomes (1979). He finds that the inheritance effect of the estate tax increases inequality through interfering with the inheritance of bequests [page 547 of Davies (1986)]. He also points out that the inheritance effect of the estate tax holds for both altruistic bequest motives and "joy of giving" bequest motives, as long as the wealth accumulation equation is linear.

The estate tax in our model has the same inheritance effect as in the previous literature with idiosyncratic labor efficiency risk. We summarize these discussions on the inheritance effect of the estate tax in Table 1.

3.2.2 The redistribution effect

The higher the estate tax ζ, the higher the term $\zeta \bar{K}$ in Eq. (16), which reflects the lump-sum transfer from the government. Thus, the inequality of $c_6 l_s + c_7 \zeta \bar{K}$ becomes

Table 2 The redistribution effect of the estate tax

Models	Bequest motives	The redistribution effect of the estate tax
This paper Bossmann et al. (2007)	"joy of giving"	To reduce the long-run wealth inequality
Becker and Tomes (1979) Davies (1986)	Altruism	To increase the long-run wealth inequality

lower.[16] The wealth distribution becomes more equal due to the redistribution effect of the estate tax.

In our model agents have "joy of giving" bequest motives and logarithmic utility functions. Estate taxes have no impact on the aggregate wealth. Thus, raising estate taxes increases government revenues and subsidies. The redistribution effect of the estate tax decreases wealth inequality. This is different from the redistribution effect of Davies (1986). Davies (1986) uses altruistic bequest motives as in Becker and Tomes (1979) and finds that the redistribution effect of increasing estate taxes usually increases wealth inequality. Numerical results of Davies (1986) show that agent's optimal reactions cause the tax base to reduce by a percentage more that the increase in the estate tax. Thus, the transfer decreases in the long run. Our model contrasts with these studies in the redistribution channel of the estate tax.

Bossmann et al. (2007) found that the form of bequest motives plays a crucial role for the impact of the estate tax on the long-run wealth inequality. However, they do not separate the two channels of the inheritance effect and the redistribution effect. Thus, they do not show that different forms of bequest motives, altruism and "joy of giving", imply different redistribution effects of the estate tax. Our paper contributes to the literature by finding this different redistribution effect of the estate tax in a model with "joy of giving" bequest motives.

We summarize these discussions on the redistribution effect of the estate tax in Table 2.

3.2.3 The net effect

The inheritance effect and the redistribution effect of the estate tax work in opposite directions in our model. We then investigate the net effect of the estate tax on the long-run wealth inequality.

We start from a static case.

Lemma 1 *For a nonnegative random variable X with a positive finite mean, if* $0 \leq \hat{\zeta} \leq \zeta < 1$, *then* $(1-\zeta)X + \zeta E(X) \succeq_L (1-\hat{\zeta})X + \hat{\zeta} E(X)$. *Thus,* $(1-\zeta)X + \zeta E(X) \preceq_{cx} (1-\hat{\zeta})X + \hat{\zeta} E(X)$.

[16] This intuition comes from the mathematical result that $X+a$ Lorenz dominates $X+b$ for any nonnegative random variable X with a finite positive mean and $a > b > 0$ [see Theorem 3.A.25 of Shaked and Shanthikumar (2010)]. Thus, $X + a$ is more equal than $X + b$.

A flat tax plus a lump-sum transfer is equivalent to a progressive tax since the effective average tax rate is increasing in wealth.[17] The higher the tax rate ζ, the higher the lump-sum transfer. And the wealth distribution becomes more equal.[18]

To highlight the main finding of our decomposition results, we only concentrate on the case of $i.i.d.$ $\{l_t\}$, when we study the net effect of the estate tax. Future work could investigate whether the Lorenz dominance result holds for the realistic case of $\{l_t\}$ which is correlated along generations.

Assumption 4 $\{l_t\}$ is $i.i.d.$

Let a_∞^ζ be the stationary wealth distribution of an economy with an estate tax ζ, and $a_\infty^{\hat{\zeta}}$ be the stationary wealth distribution of an economy with an estate tax $\hat{\zeta}$.

Theorem 2 *Under Assumptions 1–4, we have* $a_\infty^\zeta \succeq_L a_\infty^{\hat{\zeta}}$ *for* $\zeta \geq \hat{\zeta}$.

Theorem 2 extends the intuition in the static situation to stationary wealth distributions. For two economies with different estate tax rates, the economy with a higher estate tax rate has a more equal stationary wealth distribution. Bossmann et al. (2007) assume that $\{l_t\}$ either is $i.i.d.$ or follows a linear mean-reverting process. Using the coefficient of variation as their inequality measure, Bossmann et al. (2007) showed that estate taxes reduce wealth inequality.[19]

Following Davies (1986), we use a decomposition technique to investigate channels through which the estate tax influence the long-run wealth inequality. As in the previous literature with idiosyncratic labor efficiency risk, the inheritance effect of the estate tax increases the long-run wealth inequality. More importantly, we find that the redistribution effect of the estate tax decreases wealth inequality in our model. Theorem 2 shows that the redistribution effect of raising estate taxes dominates the inheritance effect. And estate taxes reduce wealth inequality in our model.

In Becker and Tomes (1979) and Davies (1986), the inheritance effect and the redistribution effect of the estate tax increase the long-run wealth inequality. These two effects work in the same direction. Thus, the net effect of increasing estate taxes on the long-run wealth distribution is disequalizing. We briefly review some main results of Becker–Tomes models in "Appendix A.8."

We summarize these discussions on the net effect of the estate tax in Table 3.

[17] For an individual with before-tax wealth x, the effective average tax rate is

$$\frac{x - [(1-\zeta)x + \zeta E(X)]}{x} = \zeta \left[1 - \frac{E(X)}{x}\right],$$

which is increasing in x.

[18] See Fellman (1976) for a study on the effect of progressive taxes on income distributions.

[19] In a simulation exercise Bossmann et al. (2007) found that the estate tax reduces the Gini coefficient of the long-run wealth distribution. Our theoretical result of Theorem 2 supports the simulation results of the Gini coefficient in Bossmann et al. (2007).

Table 3 The net effect of the estate tax

Models	Bequest motives	The net effect of the estate tax
This paper Bossmann et al. (2007)	"joy of giving"	To reduce the long-run wealth inequality
Becker and Tomes (1979) Davies (1986)	Altruism	To increase the long-run wealth inequality

These findings help us to understand the impact of estate taxes on the long-run wealth inequality. Davies (1986) found that the inheritance effect of the estate tax increases the long-run wealth inequality in the model with idiosyncratic labor efficiency risk.[20] This result holds for both altruistic bequest motives and "joy of giving" bequest motives, as long as the wealth accumulation equation is linear. Thus, the redistribution effect plays an important role in determining the net effect of the estate tax on wealth inequality. If the redistribution effect increases wealth inequality, as in Becker and Tomes (1979) and Davies (1986), then the net effect of the estate tax increases the long-run wealth inequality. If the redistribution effect decreases wealth inequality and it dominates the inheritance effect, as in our model and Bossmann et al. (2007), then the net effect of the estate tax reduces the long-run wealth inequality.

4 Housing

We introduce housing into our benchmark model. There is 1 unit of housing in the economy. We assume that housing does not depreciate.

An agent born in period t buys housing h^o_{t+1} at the end of period t and uses the housing in period $t+1$.[21] Then, the agent sells the housing at the end of period $t+1$. The young agent's problem is

$$\begin{aligned}
&\max_{c_t^y, s_t, h^o_{t+1}, c^o_{t+1}, b_{t+1}} \log c_t^y + \beta \left(\log c^o_{t+1} + \phi \log h^o_{t+1} + \chi \log \left[(1-\zeta)\, b_{t+1} \right] \right) \\
&\text{s.t. } c_t^y + s_t + p_t h^o_{t+1} \\
&\quad = w_t l_t + (1-\zeta)\, b_t + g_t, \\
&\quad c^o_{t+1} + b_{t+1} = (1 + r_{t+1}) s_t + p_{t+1} h^o_{t+1},
\end{aligned}$$

[20] Zhu (2017) introduced idiosyncratic investment risk into the Becker–Tomes model and found that the inheritance effect of the estate tax reduces the long-run wealth inequality in the model with sufficiently volatile idiosyncratic investment risk.

[21] We assume that young agents live together with their parents.

The agent's optimal policy functions are

$$
\begin{aligned}
c_{t+1}^{o} &= \frac{1}{1+\chi}\left[(1+r_{t+1})s_t + p_{t+1}h_{t+1}^{o}\right],\\
b_{t+1} &= \frac{\chi}{1+\chi}\left[(1+r_{t+1})s_t + p_{t+1}h_{t+1}^{o}\right],\\
c_t^{y} &= \frac{1}{1+\beta\left(1+\chi+\phi\right)}\left[w_t l_t + (1-\zeta)\, b_t + g_t\right],\\
s_t &= \beta\left[1+\chi-\frac{\phi}{(1+r_{t+1})\frac{p_t}{p_{t+1}}-1}\right]\frac{1}{1+\beta\left(1+\chi+\phi\right)}\\
&\quad \times\left[w_t l_t + (1-\zeta)\, b_t + g_t\right],
\end{aligned}
$$

and

$$
h_{t+1}^{o} = \frac{\beta\phi}{p_t - \frac{p_{t+1}}{1+r_{t+1}}}\frac{1}{1+\beta\left(1+\chi+\phi\right)}\left[w_t l_t + (1-\zeta)\, b_t + g_t\right].
$$

4.1 The capital market

From the government's budget constraint we have $g_t = \zeta \int b_t di$, where $\int di$ denotes the aggregation of young agents. The capital market clearing condition gives us

$$
K_{t+1} = \int s_t di,
$$

where $\int di$ denotes the aggregation of young agents.

We will concentrate on the steady-state aggregate economy in which the aggregate capital K, the wage rate w, the interest rate r, the housing price p, and the lump-sum transfer g are constant.

From the agent's policy functions we know that the aggregate capital follows

$$
K_{t+1} = \frac{\beta\left[(1+\chi)\, r-\phi\right]}{\left[1+\beta\left(1+\chi+\phi\right)\right] r}\left[w + \frac{\chi r}{(1+\chi)\, r-\phi}(1+r)K_t\right]. \tag{17}
$$

In the steady-state aggregate economy we have $K_{t+1} = K_t = K$. Thus, $w = (1-\alpha)AK^{\alpha}$ and $r = \alpha AK^{\alpha-1} - \delta$. From Eq. (17) we have

$$
\frac{1-\alpha}{\alpha}(r+\delta)\left[(1+\chi)\, r-\phi\right] + \chi r^2 - \left(1+\frac{1}{\beta}+\phi\right) r = 0, \tag{18}
$$

which determines the equilibrium interest rate r. We thus know that $r > \frac{\phi}{1+\chi}$ from Eq. (18).[22] Equation (18) does not depend on the estate tax ζ. Thus, the equilibrium

[22] The negative root of Eq. (18) cannot be the equilibrium interest rate in the economy with housing. The agent has two ways of holding assets from the young period to the old period, savings and housing.

interest rate r does not depend on the estate tax ζ. Using $r = \alpha A K^{\alpha-1} - \delta$, we have $K = \left(\frac{\alpha A}{r+\delta}\right)^{\frac{1}{1-\alpha}}$.

4.2 The housing market

The housing market clearing condition is

$$\int h^{o}_{t+1} di = 1, \tag{19}$$

where $\int di$ denotes the aggregation of old agents.

From the agent's policy functions we have

$$ph^{o}_{t+1} = \frac{(1+r)\phi}{(1+\chi)r - \phi} s_t.$$

Using the housing market clearing condition (19) we have

$$p = \frac{(1+r)\phi}{(1+\chi)r - \phi} K.$$

4.3 The wealth distribution

Let $a_{t+1} = s_t + ph^{o}_{t+1}$. Now one component of the wealth is saving. We include housing as the other component of the wealth. From the agent's policy functions we have the individual wealth accumulation equation,

$$a_{t+1} = c_9 l_t + c_{10}\left[(1-\zeta)a_t + \zeta \bar{W}\right], \tag{20}$$

where $c_9 = \frac{\beta(1+\chi+\phi)}{1+\beta(1+\chi+\phi)} w$ and $c_{10} = \frac{\beta\chi}{1+\beta(1+\chi+\phi)}(1+r)$. $\bar{W}$ denotes the aggregate wealth of the economy,

$$\bar{W} = \int a_t di = K + p.$$

Note that c_9, c_{10}, and $\bar{W}$ do not depend on the estate tax ζ.

From Eq. (20) we have the long-run wealth distribution,

$$a_\infty =_{st} \sum_{s=0}^{\infty} c_{11}^{s}\left(c_9 l_s + c_{10}\zeta \bar{W}\right),$$

Footnote 22 continued
We assume that housing does not depreciate. And the housing price does not change in the stationary equilibrium. Thus, the return of housing is positive. There exist arbitrages if the interest rate of saving is negative.

where $c_{11} = c_{10}(1-\zeta)$.

Comparing Eqs. (20) and (15) we find that all the theoretical results of the long-run wealth inequality in the benchmark model still hold in a model with housing. We can decompose the impact of the estate tax on the long-run wealth inequality into two channels, the inheritance effect and the redistribution effect. The channel through which the estate tax influences c_{11} is the inheritance effect. The higher the ζ, the lower the c_{11}. Thus, the inheritance effect increases the long-run wealth inequality. The channel through which the estate tax influences the term $c_9 l_s + c_{10}\zeta \bar{W}$ is the redistribution effect. The higher the ζ, the higher the $\zeta \bar{W}$. Thus, the redistribution effect reduces the long-run wealth inequality. As in Theorem 2, the net effect is that the estate tax ζ reduces the long-run wealth inequality.

5 A life cycle model

We investigate the stationary distribution of individual wealth accumulation process in the steady-state aggregate economy. We will concentrate on the steady-state aggregate economy in which the aggregate capital K, the wage rate w, the interest rate r, and the lump-sum transfer g are constant.

We normalize the population of the economy to 1. Agents live for $T+1$ periods. At the end of age $T+1$, the agent dies and gives birth to one child. The retirement age is R. At the beginning of the life, the agent of dynasty n draws his labor efficiency l_n. Then, the agent keeps this labor efficiency for the whole life. We assume that $\{l_n\}$ is $i.i.d.$ along generations. The agent's problem is

$$\begin{aligned}
&\max_{c_t^y, s_t, h_{t+1}^o, c_{t+1}^o, b_{t+1}} \sum_{\tau=0}^{T} \beta^\tau \frac{(c_{n,\tau})^{1-\eta}-1}{1-\eta} + \beta^T \chi \frac{\left[(1-\zeta)\, b_{n+1}\right]^{1-\eta}-1}{1-\eta} \\
&\text{s.t. } a_{n,1} + c_{n,0} = w l_n + (1-\zeta)\, b_n + g, \\
&a_{n,\tau+1} + c_{n,\tau} = (1+r) a_{n,\tau} + w l_n, \ 1 \le \tau \le R-1, \\
&a_{n,\tau+1} + c_{n,\tau} = (1+r) a_{n,\tau}, \ R \le \tau \le T-1, \\
&c_{n,T} + b_{n+1} = (1+r) a_{n,T}.
\end{aligned}$$

The agent's optimal policy functions during the retirement periods are

$$\begin{aligned}
b_{n+1} &= \frac{\chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}}}{1+\chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}}} (1+r) a_{n,T}, \\
c_{n,T} &= \frac{1}{1+\chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}}} (1+r) a_{n,T}. \\
a_{n,\tau+1} &= \frac{\tilde{\beta}_{\tau+1}^{\frac{1}{\eta}}}{1+\tilde{\beta}_{\tau+1}^{\frac{1}{\eta}}} (1+r) a_{n,\tau}, \ R \le \tau \le T-1,
\end{aligned}$$

and

$$c_{n,\tau} = \frac{1}{1+\tilde{\beta}_{\tau+1}^{\frac{1}{\eta}}}(1+r)a_{n,\tau}, \ R \leq \tau \leq T-1,$$

where $\tilde{\beta}_T = \beta \left[1 + \chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}} \right]^{\eta} (1+r)^{1-\eta}$ and $\tilde{\beta}_\tau = \beta \left(1 + \tilde{\beta}_{\tau+1}^{\frac{1}{\eta}} \right)^{\eta} (1+r)^{1-\eta}$ for $R \leq \tau \leq T-1$.

The agent's optimal policy functions during the work periods are

$$\begin{aligned}
c_{n,0} &= \frac{1}{1+\tilde{\beta}_1^{\frac{1}{\eta}}} \left[(1-\zeta)\, b_n + g + \left(1+\frac{1}{r}\right)\left(1-\frac{1}{(1+r)^R}\right) w l_n \right], \\
a_{n,1} &= (1-\zeta)\, b_n + g + w l_n - c_{n,0}, \\
c_{n,\tau} &= \frac{1+r}{1+\tilde{\beta}_{\tau+1}^{\frac{1}{\eta}}} \left[a_{n,\tau} + \frac{1}{r}\left(1-\frac{1}{(1+r)^{R-\tau}}\right) w l_n \right], \ 1 \leq \tau \leq R-1,
\end{aligned}$$

and

$$a_{n,\tau+1} = (1+r)a_{n,\tau} + w l_n - c_{n,\tau}, \ 1 \leq \tau \leq R-1,$$

where $\tilde{\beta}_\tau = \beta \left(1 + \tilde{\beta}_{\tau+1}^{\frac{1}{\eta}} \right)^{\eta} (1+r)^{1-\eta}$ for $1 \leq \tau \leq R-1$.

From the government's budget constraint we have

$$g = \zeta \int b_n di,$$

where $\int di$ denotes the aggregation of age 0 agents.

The capital market clearing condition gives us

$$K = \frac{1}{T} \sum_{\tau=1}^{T} \int a_\tau di,$$

where $\int a_\tau di$ denotes the aggregate capital within the age τ cohort.

The aggregate population of workers is 1. And we assume that $E(l_n) = 1$. Thus, the labor market clearing condition is

$$L = \frac{R}{T}.$$

In the steady-state aggregate economy we have

$$w = (1-\alpha) A K^{\alpha} L^{-\alpha},$$

and

$$r = \alpha A K^{\alpha-1} L^{1-\alpha} - \delta.$$

Table 4 The impact of the estate tax on the long-run wealth inequality

ζ	K	g	$\int b_n di$	Gini
0	5.440	0	0.691	0.525
0.1	5.452	0.073	0.727	0.523
0.2	5.476	0.155	0.769	0.520
0.3	5.506	0.247	0.819	0.518
0.4	5.541	0.352	0.881	0.514
0.5	5.596	0.479	0.957	0.510
0.6	5.651	0.638	1.064	0.505

To illustrate the impact of the estate tax ζ on the long-run wealth inequality, we implement a simple calibration exercise. We pick $T = 60$, $R = 40$, $\beta = 0.95$, $\eta = 2$, $\chi = 0.8$, $A = 1$, $\alpha = \frac{1}{3}$, and $\delta = 0.05$. We assume that $l_t \sim U[0, 2]$.[23] Thus, $E(l_t) = 1$. Table 4 reports the simulation results of the life cycle model.

Table 4 shows that the higher the estate tax ζ, the lower the Gini coefficient of the long-run wealth distribution. The estate tax reduces the long-run wealth inequality. Table 4 also shows that the higher the estate tax ζ, the higher the g. Thus, the redistribution effect of the estate tax decreases the long-run wealth inequality. The results of the long-run wealth inequality in our benchmark model are still true in this life cycle model.

6 Conclusion

Bossmann et al. (2007) found that estate taxes reduce the long-run wealth inequality. This result contrasts with the findings of the previous literature with idiosyncratic labor efficiency risk, such as Becker and Tomes (1979) and Davies (1986). These papers show that estate taxes usually increase the long-run wealth inequality. In this paper we use the decomposition technique developed by Davies (1986) to reinvestigate the impact of estate taxes on the long-run wealth inequality. We find that the redistribution effect plays an important role in determining the effect of the estate tax on the long-run wealth inequality. We also extend our benchmark model in two directions. In the first extension, we include housing as a new asset in the model. In the other extension, we permit the agent to live for more than two periods. In these extensions we show that the results of the long-run wealth inequality in our benchmark model are still true.

Our findings also help us to understand how different ways of modeling bequest motives influence the impact of estate taxes on wealth inequality. Different forms of bequest motives, altruism and “joy of giving”, do not influence the inheritance effect of the estate tax, but they imply different redistribution effects of the estate tax. Thus, different forms of bequest motives influence the impact of estate taxes on wealth inequality through the redistribution effect.

[23] The Gini coefficient of the earnings distribution is 0.33. The Gini coefficients of the long-run wealth distribution in Table 4 are larger than this number since there exists a life cycle pattern of savings.

A Appendix

A.1 Proof of Proposition 1

Proof Letting $K_{t+1} = K_t = K$ in Eq. (11) we have

$$K = \left(\frac{1 - \alpha + \varphi\alpha}{1 + \tilde{\beta}^{-\frac{1}{\eta}} - \varphi(1 - \delta)} A \right)^{\frac{1}{1-\alpha}}, \tag{A.1}$$

where $\tilde{\beta} = \beta \left[1 + \chi^{\frac{1}{\eta}} (1 - \zeta)^{\frac{1-\eta}{\eta}} \right]^{\eta} (1 + r)^{1-\eta}$ and $r = \alpha A K^{\alpha-1} - \delta$.

Plugging Eq. (A.1) into $r = \alpha A K^{\alpha-1} - \delta$ we have

$$\frac{r + \delta}{\alpha} = \frac{1 + \tilde{\beta}^{-\frac{1}{\eta}} - \varphi(1 - \delta)}{1 - \alpha + \varphi\alpha}. \tag{A.2}$$

Plugging $\tilde{\beta} = \beta \left[1 + \chi^{\frac{1}{\eta}} (1 - \zeta)^{\frac{1-\eta}{\eta}} \right]^{\eta} (1 + r)^{1-\eta} = \frac{\beta}{(1-\varphi)^{\eta}} (1 + r)^{1-\eta}$ into Eq. (A.2) we have

$$\frac{1 - \alpha}{\alpha} (r + \delta) + \varphi(1 + r) - (1 - \varphi)\beta^{-\frac{1}{\eta}} (1 + r)^{1-\frac{1}{\eta}} = 1.$$

We show Proposition 1 in two cases:

Case (i) $\eta > 1$

Note that $0 < \varphi < 1$. Define

$$h(\varphi, r) = \frac{1 - \alpha}{\alpha} (r + \delta) + \varphi(1 + r) - (1 - \varphi)\beta^{-\frac{1}{\eta}} (1 + r)^{1-\frac{1}{\eta}}.$$

The equilibrium r is determined by

$$h(\varphi, r) = 1.$$

Note that $h(\varphi, r)$ is a continuous function of r, with

$$h(\varphi, -\delta) = \varphi(1 - \delta) - (1 - \varphi)\beta^{-\frac{1}{\eta}} (1 - \delta)^{1-\frac{1}{\eta}} < \varphi(1 - \delta) < 1$$

and

$$\lim_{r \to \infty} h(\varphi, r) = \infty$$

Also $h_{22}(\varphi, r) = \left(1 - \frac{1}{\eta}\right) \frac{1}{\eta} (1 - \varphi)\beta^{-\frac{1}{\eta}} (1 + r)^{-\frac{1}{\eta}-1} > 0$ due to $\eta > 1$. Thus, $h(\varphi, r)$ is a strictly convex function of r. Therefore, there must exist a unique equilibrium $r > -\delta$.[24]

[24] In the equilibrium r could be negative. Since saving is the only way to bring wealth to the next period, even if r is negative, the agent still saves.

Note that $h(\varphi, r)$ is strictly increasing in φ. For $\varphi_1 < \varphi_2 < 1$, suppose that

$$h(\varphi_1, r_1) = 1 \text{ and } h(\varphi_2, r_2) = 1.$$

We have

$$h(\varphi_2, r_1) > h(\varphi_1, r_1) = 1.$$

Thus, $r_2 < r_1$ since $h(\varphi_2, -\delta) < 1$ and $h(\varphi_2, r)$ is a continuous function of r. A higher χ implies a higher φ . Thus, a higher χ implies a lower r and a higher K.

Case (ii) $\eta = 1$

In this case $\tilde{\beta} = \beta(1+\chi)$ and $\varphi = \frac{1}{1+\frac{1}{\chi}}$, Eq. (A.1) implies

$$\begin{aligned} K &= \left(\frac{1-\alpha+\chi}{1+\frac{1}{\beta}+\delta\chi} A \right)^{\frac{1}{1-\alpha}} \\ &= \left(\left[\frac{1}{\delta} - \frac{\frac{1}{\delta}\left(1+\frac{1}{\beta}\right) - (1-\alpha)}{1+\frac{1}{\beta}+\delta\chi} \right] A \right)^{\frac{1}{1-\alpha}}. \end{aligned}$$

Thus, a higher χ implies a higher K. □

A.2 Proof of Proposition 2

Proof Obviously $c_4 \geq 0$. From Eq. (A.2) we have

$$1 + r = \frac{\left(1 + \tilde{\beta}^{-\frac{1}{\eta}}\right)\alpha + (1-\delta)(1-\alpha)}{(1-\alpha) + \varphi\alpha}$$

Thus,

$$c_4 = \frac{(1-\zeta)\,\varphi(1+r)}{1+\tilde{\beta}^{-\frac{1}{\eta}}} = (1-\zeta)\,\frac{\alpha + \frac{1-\delta}{1+\tilde{\beta}^{-\frac{1}{\eta}}}(1-\alpha)}{\alpha + \frac{1}{\varphi}(1-\alpha)} < 1$$

since $\tilde{\beta} = \beta\left[1 + \chi^{\frac{1}{\eta}}(1-\zeta)^{\frac{1-\eta}{\eta}}\right]^{\eta}(1+r)^{1-\eta} > 0$ and $0 < \varphi < 1$. □

A.3 Proof of Proposition 3

Proof From Eq. (13) we have

$$a_{t+1} = c_3 l_t + c_4 a_t + c_5,$$

where $c_3 = \frac{1}{1+\tilde{\beta}^{-\frac{1}{\eta}}} w$, $c_4 = \frac{(1-\zeta)\varphi(1+r)}{1+\tilde{\beta}^{-\frac{1}{\eta}}}$, and $c_5 = \frac{\zeta\varphi(1+r)}{1+\tilde{\beta}^{-\frac{1}{\eta}}} K$. Let $B_t = c_5 + c_3 l_t$. We have

$$a_{t+1} = c_4 a_t + B_t. \tag{A.3}$$

Note that $\{B_t\}$ is stationary and ergodic since $\{l_t\}$ is stationary and ergodic by Assumption 1. We have $-\infty \le \log c_4 < 0$. Also $E(B_t) = c_5 + c_3 < \infty$, since $E(l_t) = 1$ by Assumption 2. Thus, $E(\log B_t)^+ \le E(B_t) < \infty$. By Theorem 1 of Brandt (1986) we know that a_t converges to $\sum_{j=0}^{\infty} c_4^j B_{t-j-1}$ almost surely as t approaches infinity. Thus, we have

$$a_t \to_{st} \sum_{j=0}^{\infty} c_4^j B_{t-j-1} \text{ as } t \to \infty.$$

Since $\{B_t\}$ is stationary, we know that the sequence of $(B_{t-1}, B_{t-2}, \ldots, B_{t-j-1}, \ldots)$ has the same distribution as the sequence of $(B_0, B_1, \ldots, B_s, \ldots)$. Thus, we have

$$\sum_{j=0}^{\infty} c_4^j B_{t-j-1} =_{st} \sum_{s=0}^{\infty} c_4^s B_s, \ \forall t \in \mathbb{Z}.$$

Let

$$a_\infty =_{st} \sum_{s=0}^{\infty} c_4^s B_s =_{st} c_3 \sum_{s=0}^{\infty} c_4^s l_s + \frac{c_5}{1-c_4}.$$

Thus, we know that

$$a_t \to_{st} a_\infty \text{ as } t \to \infty.$$

□

A.4 Proof of Theorem 1

Proof Theorem 3.A.36 of Shaked and Shanthikumar (2010) shows that

Lemma 2 *Let $X_1, X_2, \ldots, X_n$ and Y be $n+1$ random variables. If $X_i \preceq_{cx} Y$, $i = 1, 2, \ldots, n$, then*

$$\sum_{i=1}^{n} a_i X_i \preceq_{cx} Y,$$

whenever $a_i \ge 0$, $i = 1, 2, \ldots, n$, and $\sum_{i=1}^{n} a_i = 1$.[25]

Theorem 3.A.10 of Shaked and Shanthikumar (2010) states that

[25] For two random variables X and Y, X is smaller than Y in the convex order, denoted by $X \preceq_{cx} Y$, if and only if

$$E[\phi(X)] \le E[\phi(Y)]$$

for all convex functions $\phi : \mathbb{R} \to \mathbb{R}$, provided the expectations exist. For more properties of the convex order, see Shaked and Shanthikumar (2010).

Lemma 3 *Let X and Y be two nonnegative random variables with equal means. Then, $X \preceq_{cx} Y$ if and only if $L_X(p) \geq L_Y(p)$ for all $p \in [0, 1]$.*

Note that a_∞^A has the same Lorenz curve as l_1. We only need to show that $a_\infty^B \succeq_L l_1$. In economy B, pick $a_1 = \frac{c_3}{1-c_4}$.[26] Thus,

$$a_1 \preceq_{cx} \frac{c_3}{1-c_4} l_1$$

since $a_1 = E\left(\frac{c_3}{1-c_4} l_1\right)$.[27]

Suppose that

$$a_t \preceq_{cx} \frac{c_3}{1-c_4} l_1.$$

Thus, $\frac{1-c_4}{c_3} a_t \preceq_{cx} l_1$.[28]

And

$$\begin{aligned} a_{t+1} &= c_3 l_t + c_4 a_t \\ &= \frac{c_3}{1-c_4}\left((1-c_4) l_t + c_4 \frac{1-c_4}{c_3} a_t\right). \end{aligned}$$

Note that $(1-c_4) l_t + c_4 \frac{1-c_4}{c_3} a_t$ is a weighted average of l_t and $\frac{1-c_4}{c_3} a_t$. For $\forall t \geq 1$, l_t and l_1 have the same distribution. We have $l_t \preceq_{cx} l_1$, $\forall t \geq 1$. By Lemma 2 we have

$$(1-c_4) l_t + c_4 \frac{1-c_4}{c_3} a_t \preceq_{cx} l_1.$$

Thus,

$$a_{t+1} \preceq_{cx} \frac{c_3}{1-c_4} l_1.$$

By mathematical induction we have

$$a_t \preceq_{cx} \frac{c_3}{1-c_4} l_1, \quad \forall t \geq 1.$$

Since $a_t \to_{st} a_\infty^B$ as t approaches infinity, we have

$$a_\infty^B \preceq_{cx} \frac{c_3}{1-c_4} l_1,$$

by part (c) of Theorem 3.A.12 of Shaked and Shanthikumar (2010). By Lemma 3 we have $a_\infty^B \succeq_L \frac{c_3}{1-c_4} l_1$ since $E\left(a_\infty^B\right) = E\left(\frac{c_3}{1-c_4} l_1\right) = \frac{c_3}{1-c_4}$. Thus, $a_\infty^B \succeq_L l_1$. □

[26] We abuse notations a little bit. We use a_t instead of a_t^B without confusions.

[27] Let X be a random variable with a finite mean. $E(X) \preceq_{cx} X$ can be established by applying Jensen's Inequality and the definition of the convex order.

[28] $X \preceq_{cx} Y$ implies $bX \preceq_{cx} bY$ for any $b \in \mathbb{R}$. Note that $\phi(bx)$ is a convex function of $x \in \mathbb{R}$ if $\phi(x)$ is a convex function of $x \in \mathbb{R}$.

A.5 Proof of Lemma 1

Proof Let

$$g(x) = (1-\zeta)x + \zeta E(X), \quad x \in [0, +\infty)$$

and

$$h(x) = (1-\hat{\zeta})x + \hat{\zeta} E(X), \quad x \in [0, +\infty)$$

Note that $g(\cdot)$ and $h(\cdot)$ are nonnegative increasing functions defined on $[0, +\infty)$, since $0 \le \hat{\zeta} \le \zeta < 1$. Also $g(x) > 0$ and $h(x) > 0$ for $x > 0$. Note that $\frac{h(x)}{g(x)}$ is increasing in $x \in (0, +\infty)$, since

$$\begin{aligned}\frac{h(x)}{g(x)} &= \frac{(1-\hat{\zeta})x + \hat{\zeta} E(X)}{(1-\zeta)x + \zeta E(X)} \\ &= \frac{1-\hat{\zeta}}{1-\zeta}\left[1 - \frac{\zeta - \hat{\zeta}}{\left(1-\hat{\zeta}\right)(1-\zeta)} \frac{E(X)}{x + \frac{\zeta}{1-\zeta}E(X)}\right].\end{aligned}$$

By Theorem 3.A.26 of Shaked and Shanthikumar (2010) we have $g(X) \succeq_L h(X)$, i.e., $(1-\zeta)X + \zeta E(X) \succeq_L (1-\hat{\zeta})X + \hat{\zeta} E(X)$. By Lemma 3 we have $(1-\zeta)X + \zeta E(X) \preceq_{cx} (1-\hat{\zeta})X + \hat{\zeta} E(X)$ since $E[(1-\zeta)X + \zeta E(X)] = E(X) = E\left[(1-\hat{\zeta})X + \hat{\zeta} E(X)\right]$. □

A.6 Proof of Theorem 2

Proof Note that a_∞^ζ is the stationary distribution of the stochastic process $\{a_t^\zeta\}$ which is generated by

$$a_{t+1}^\zeta = c_6 l_t + c_7\left[(1-\zeta)a_t^\zeta + \zeta \bar{K}\right]$$

and a given a_1^ζ. And $a_\infty^{\hat{\zeta}}$ is the stationary distribution of the stochastic process $\{a_t^{\hat{\zeta}}\}$ which is generated by

$$a_{t+1}^{\hat{\zeta}} = c_6 l_t + c_7\left[(1-\hat{\zeta})a_t^{\hat{\zeta}} + \hat{\zeta} \bar{K}\right]$$

and a given $a_1^{\hat{\zeta}}$.

Let $a_1^\zeta =_{st} a_1^{\hat{\zeta}}$. Thus, $a_1^\zeta \preceq_{cx} a_1^{\hat{\zeta}}$ by the definition of the convex order.

Now suppose that $a_t^\zeta \preceq_{cx} a_t^{\hat{\zeta}}$. By Lemma 1 we have

$$(1-\zeta)a_t^\zeta + \zeta\bar{K} \preceq_{cx} (1-\hat{\zeta})a_t^\zeta + \hat{\zeta}\bar{K}$$

since $E\left(a_t^\zeta\right) = \bar{K}$.

By Corollary 3.A.22 of Shaked and Shanthikumar (2010) we have $(1-\hat{\zeta})a_t^{\zeta} \preceq_{cx} (1-\hat{\zeta})a_t^{\hat{\zeta}}$ since $\left(1-\hat{\zeta}\right)$ is independent of a_t^{ζ} and $a_t^{\hat{\zeta}}$. By Part (d) of Theorem 3.A.12 of Shaked and Shanthikumar (2010) we have

$$(1-\hat{\zeta})a_t^{\zeta} + \hat{\zeta}\bar{K} \preceq_{cx} (1-\hat{\zeta})a_t^{\hat{\zeta}} + \hat{\zeta}\bar{K},$$

since $\hat{\zeta}\bar{K}$ is independent of $(1-\hat{\zeta})a_t^{\zeta}$ and $(1-\hat{\zeta})a_t^{\hat{\zeta}}$. By the transitivity of the convex order we have

$$(1-\zeta)a_t^{\zeta} + \zeta\bar{K} \preceq_{cx} (1-\hat{\zeta})a_t^{\hat{\zeta}} + \hat{\zeta}\bar{K}.$$

Thus, we have $c_7\left[(1-\zeta)a_t^{\zeta} + \zeta\bar{K}\right] \preceq_{cx} c_7\left[(1-\hat{\zeta})a_t^{\hat{\zeta}} + \hat{\zeta}\bar{K}\right]$ by the property of the convex order in Footnote 28. Note that $c_6 l_t$ and $c_7\left[(1-\zeta)a_t^{\zeta} + \zeta\bar{K}\right]$ are independent. And $c_6 l_t$ and $c_7\left[(1-\hat{\zeta})a_t^{\hat{\zeta}} + \hat{\zeta}\bar{K}\right]$ are independent. Thus, by part (d) of Theorem 3.A.12 of Shaked and Shanthikumar (2010), we have

$$c_6 l_t + c_7\left[(1-\zeta)a_t^{\zeta} + \zeta\bar{K}\right] \preceq_{cx} c_6 l_t + c_7\left[(1-\hat{\zeta})a_t^{\hat{\zeta}} + \hat{\zeta}\bar{K}\right].$$

Thus, we have

$$a_{t+1}^{\zeta} \preceq_{cx} a_{t+1}^{\hat{\zeta}}.$$

By mathematical induction we have

$$a_t^{\zeta} \preceq_{cx} a_t^{\hat{\zeta}}, \ \forall t \geq 1.$$

Since $a_t^{\zeta} \rightarrow_{st} a_{\infty}^{\zeta}$ and $a_t^{\hat{\zeta}} \rightarrow_{st} a_{\infty}^{\hat{\zeta}}$ as t approaches infinity, we have

$$a_{\infty}^{\zeta} \preceq_{cx} a_{\infty}^{\hat{\zeta}},$$

by part (c) of Theorem 3.A.12 of Shaked and Shanthikumar (2010). By Lemma 3 we have

$$a_{\infty}^{\zeta} \succeq_L a_{\infty}^{\hat{\zeta}},$$

since $E\left(a_{\infty}^{\zeta}\right) = E\left(a_{\infty}^{\hat{\zeta}}\right) = \bar{K}$. □

A.7 An alternative setup of the model

Here we investigate an alternative setup of our benchmark model. The main difference is that b_t in our benchmark model is the before-tax bequest. In this alternative setup, b_t is the after-tax bequest.

The agent's problem is

$$\max_{c_t^y, s_t, c_{t+1}^o, b_{t+1}} \log c_t^y + \beta \left(\log c_{t+1}^o + \chi \log b_{t+1} \right)$$
$$\text{s.t. } c_t^y + s_t = w_t l_t + b_t + g_t,$$
$$c_{t+1}^o + (1+\zeta)\, b_{t+1} = (1+r_{t+1}) s_t.$$

The agent's optimal policy functions are

$$c_{t+1}^o = \frac{1}{1+\chi}(1+r_{t+1}) s_t,$$
$$b_{t+1} = \frac{\chi}{(1+\chi)(1+\zeta)}(1+r_{t+1}) s_t,$$
$$c_t^y = \frac{1}{1+\beta(1+\chi)}\left(w_t l_t + b_t + g_t\right),$$

and

$$s_t = \frac{\beta(1+\chi)}{1+\beta(1+\chi)}\left(w_t l_t + b_t + g_t\right).$$

From the government's budget constraint we have

$$g_t = \zeta \int b_t di,$$

where $\int di$ denotes the aggregation of old agents.

Thus, the aggregate capital follows

$$K_{t+1} = \int s_t di$$
$$= \frac{\beta(1+\chi)}{1+\beta(1+\chi)}\left[w_t + \frac{\chi}{1+\chi}(1+r_t) K_t \right],$$

where $w_t = (1-\alpha) A K_t^{\alpha}$ and $r_t = \alpha A K_t^{\alpha-1} - \delta$.

In the steady-state aggregate economy we have $K_{t+1} = K_t = \bar{K}$. Thus, we have

$$\bar{K} = \left(\frac{1-\alpha+\chi}{1+\frac{1}{\beta}+\delta\chi} A \right)^{\frac{1}{1-\alpha}}.$$

The estate tax does not affect the aggregate capital. Then, $\bar{w} = (1-\alpha) A \left(\bar{K}\right)^{\alpha}$ and $\bar{r} = \alpha A \left(\bar{K}\right)^{\alpha-1} - \delta$.

Let $a_{t+1} = s_t$. From the agent's policy functions we have the individual wealth accumulation equation,

$$a_{t+1} = c_6 l_t + c_7 \left[\frac{1}{1+\zeta} a_t + \frac{\zeta}{1+\zeta} \bar{K} \right], \tag{21}$$

with $c_6 = \frac{1}{1+\frac{1}{\beta(1+\chi)}}\bar{w}$ and $c_7 = \frac{1}{\left(1+\frac{1}{\beta(1+\chi)}\right)\left(1+\frac{1}{\chi}\right)}(1+\bar{r})$. Both c_6 and c_7 do not depend on the estate tax ζ.

From Eq. (21) we have the long-run wealth distribution,

$$a_\infty =_{st} \sum_{s=0}^{\infty} (\tilde{c}_8)^s \left(c_6 l_s + c_7 \frac{\zeta}{1+\zeta}\bar{K}\right),$$

where $\tilde{c}_8 = c_7 \frac{1}{1+\zeta}$. Comparing Eqs. (21) and (15) we find that all the theoretical results of the long-run wealth inequality in the benchmark model still hold in this alternative setup.

A.8 The Becker–Tomes model

Here we briefly review some main results of Becker–Tomes models by Becker and Tomes (1979) and Davies (1986). As in Becker and Tomes (1979) and Davies (1986), we assume that each agent only lives for one period. At the end of the period, the agent dies and gives birth to one child. The prices of r and w are exogenous and constant. Davies (1986) explained that the aim of using exogenous prices of r and w in his paper is exactly close to the general equilibrium effect of the estate tax.[29]

As in Becker and Tomes (1979) and Davies (1986) we assume that the agent can correctly anticipate the labor efficiency of his child. The agent's problem is

$$\begin{aligned} &\max_{c_t, b_{t+1}, I_{t+1}} \frac{c_t^{1-\eta}-1}{1-\eta} + \chi \frac{I_{t+1}^{1-\eta}-1}{1-\eta} \\ &\text{s.t. } c_t + b_{t+1} = I_t, \\ &I_{t+1} = w l_{t+1} + (1+r)(1-\zeta) b_{t+1} + g, \end{aligned}$$

where I_{t+1} is the total wealth of the child. The agent's optimal policy functions are

$$c_t = \frac{1}{1+[(1+r)(1-\zeta)]^{\frac{1-\eta}{\eta}} \chi^{\frac{1}{\eta}}} \left(I_t + \frac{w l_{t+1} + g}{(1+r)(1-\zeta)}\right),$$

$$b_{t+1} = \frac{1}{1+[(1+r)(1-\zeta)]^{\frac{\eta-1}{\eta}} \chi^{-\frac{1}{\eta}}} I_t - \frac{1}{1+[(1+r)(1-\zeta)]^{\frac{1-\eta}{\eta}} \chi^{\frac{1}{\eta}}} \frac{w l_{t+1} + g}{(1+r)(1-\zeta)},$$

[29] After we solve the general equilibrium in our benchmark model with "joy of giving" bequest motives, the estate tax does not affect the prices of r and w when utility functions are logarithmic. Thus, the estate tax does not have a general equilibrium effect. However, the estate tax does affect the prices of r and w in a model with altruistic bequest motives even for logarithmic utility functions. Then, the estate tax does have a general equilibrium effect. Thus, a model with altruistic bequest motives and endogenous prices of r and w is not comparable to our benchmark model.
We then decide to follow the studies of Becker and Tomes (1979) and Davies (1986) to assume that the prices of r and w are exogenous. Thus, we can concentrate on the inheritance effect and the redistribution effect of the estate tax.

and

$$I_{t+1} = \frac{(1+r)(1-\zeta)}{1+[(1+r)(1-\zeta)]^{\frac{\eta-1}{\eta}}\chi^{-\frac{1}{\eta}}}\left(I_t + \frac{wl_{t+1}+g}{(1+r)(1-\zeta)}\right). \tag{22}$$

From the government's budget constraint we have

$$g = \zeta(1+r)\int b_t di,$$

where $\int di$ denotes the aggregation of young agents.

In the steady-state aggregate economy we have $\int I_{t+1}di = \int I_t di = \bar{I}$. Thus, we have

$$\bar{I} = \int I_t di = w + (1+r)(1-\zeta)\int b_t di + g = w + \frac{g}{\zeta}. \tag{23}$$

From Eq. (22) we have

$$\bar{I} = \frac{(1+r)(1-\zeta)}{1+[(1+r)(1-\zeta)]^{\frac{\eta-1}{\eta}}\chi^{-\frac{1}{\eta}}}\left(\bar{I} + \frac{w+g}{(1+r)(1-\zeta)}\right). \tag{24}$$

Combining Eqs. (23) and (24) we have

$$\bar{I} = \frac{1}{1-(1+r)\left(1-[(1+r)(1-\zeta)]^{-\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}\right)}w,$$

and

$$g = mw,$$

where $m = \frac{\zeta}{\frac{1}{(1+r)\left(1-[(1+r)(1-\zeta)]^{-\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}\right)}-1}$.

From Eq. (22) we have the individual wealth accumulation equation,

$$I_{t+1} = c_{12}I_t + \frac{1}{1+[(1+r)(1-\zeta)]^{\frac{\eta-1}{\eta}}\chi^{-\frac{1}{\eta}}}(wl_{t+1}+g), \tag{25}$$

where $c_{12} = \frac{(1+r)(1-\zeta)}{1+[(1+r)(1-\zeta)]^{\frac{\eta-1}{\eta}}\chi^{-\frac{1}{\eta}}}$.

For simplicity we assume that $\{l_t\}$ is $i.i.d$. Thus, we have

$$\mathrm{Var}(I_t) = \frac{c_{12}^2\mathrm{Var}(l_t)}{\left(1-c_{12}^2\right)[(1+r)(1-\zeta)]^2}w^2.$$

The impact of ζ on c_{12} in the individual wealth accumulation Eq. (25) represents the inheritance effect of the estate tax on the stationary wealth distribution. The higher

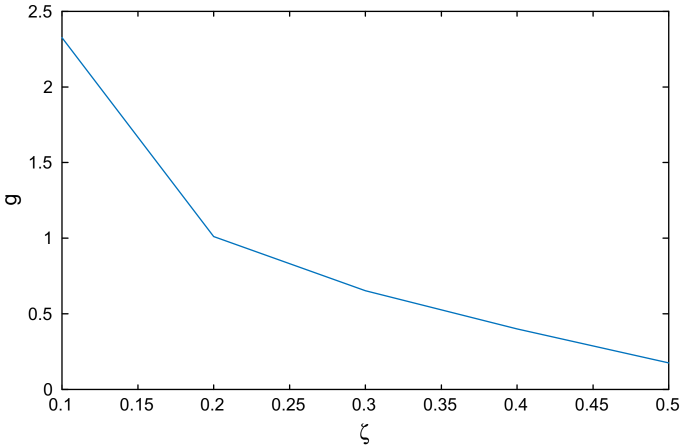

Fig. 2 The impact of the estate tax on the transfer

the estate tax ζ, the lower the c_{12}.[30] Thus, the inheritance effect of the estate tax increases the long-run wealth inequality. The impact of ζ on the lump-sum transfer g in the individual wealth accumulation Eq. (25) represents the redistribution effect of the estate tax.

We can calculate the coefficient of variation,

$$\begin{aligned} &\mathrm{CV}(I_t) \\ &= \frac{\sqrt{\mathrm{Var}(I_t)}}{\bar{I}} \\ &= \frac{1-(1+r)\left(1-[(1+r)(1-\zeta)]^{-\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}\right)}{(1+r)(1-\zeta)} \frac{c_{12}}{\sqrt{1-c_{12}^2}}\sqrt{\mathrm{Var}(l_t)}. \end{aligned}$$

To illustrate the impact of the estate tax ζ on the lump-sum transfer g and that of the estate tax ζ on the long-run wealth inequality, we implement a simple calibration exercise. We pick $\eta = 2$, $\chi = 0.8$, $r = 2$, and $w = 1$. We assume that one generation lasts for 30 years. Thus, $r = 1$ corresponds to the annual interest rate of 3.7%. We increase the estate tax ζ from 0.1 to 0.5. Figure 2 shows that the higher the estate tax ζ, the lower the g. Thus, the redistribution effect of the estate tax increases the long-run wealth inequality.

We also investigate the net effect of the estate tax on the long-run wealth inequality. We assume that $l_t \sim U[0, 2]$. Thus, $E(l_t) = 1$ and $\mathrm{Var}(l_t) = \frac{2}{3}$. Figure 3 shows that the higher the estate tax ζ, the higher the CV of the long-run wealth inequality.

[30] Note that

$$c_{12} = \frac{1}{[(1+r)(1-\zeta)]^{-1} + [(1+r)(1-\zeta)]^{-\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}}.$$

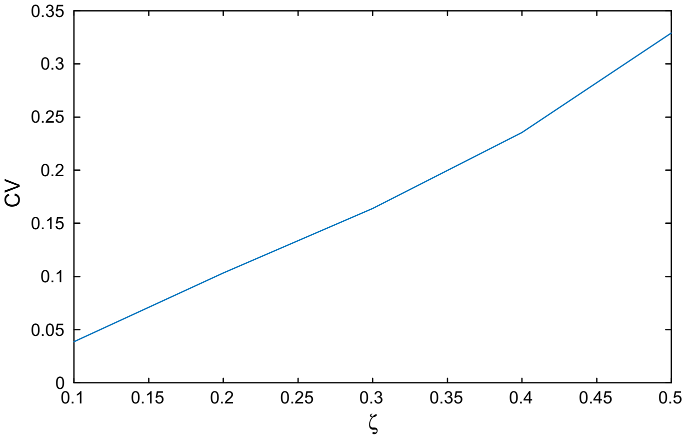

Fig. 3 The impact of the estate tax on the CV

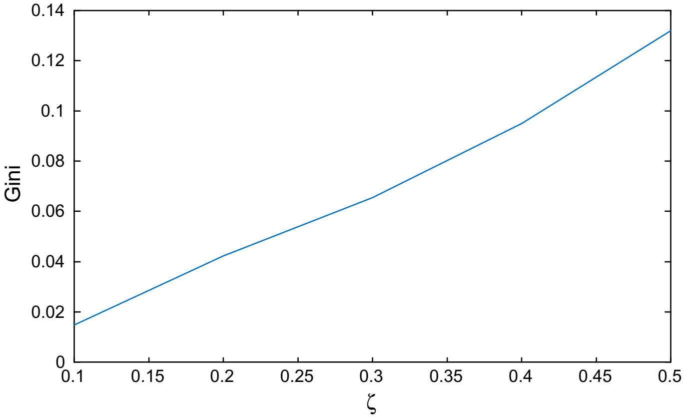

Fig. 4 The impact of the estate tax on the Gini coefficient

Figure 4 shows that the higher the estate tax ζ, the higher the Gini coefficient of the long-run wealth inequality. Figures 3 and 4 show that the estate tax increases the long-run wealth inequality. This result is reasonable since both the inheritance effect and the redistribution effect of the estate tax increase the long-run wealth inequality.

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