Tax Progressivity and the Pareto Tail of Income Distributions

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Abstract

In a continuous-time version of the Bewley-Huggett-Aiyagari model, this paper shows theoretically and numerically that the fatness of the Pareto upper tail of the income distribution depends on tax progressivity only through the general equilibrium effect on the interest rate. With confiscatory taxes (marginal income tax rate approaching 100% at the top), the Pareto exponent is independent of tax progressivity.

JEL classifications: D31, E21, H31
Keywords: Tax progressivity; Pareto tail; Bewley-Huggett-Aiyagari model

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1 Introduction

Will a higher or lower progressivity of income taxation alter the fatness of the Pareto upper tail of income distributions? Perhaps surprisingly, our answer is negative. We demonstrate this result theoretically and numerically in a continuous-time version of the Bewley-Huggett-Aiyagari model in which idiosyncratic risk includes both earnings and investment.[1]

We assume that the tax scheme is imposed on the sum of different types of personal income and, specifically, it does not discriminate between capital and labor income.[2] Piketty et al. (2022) argued that the fuzziness of the capital vs. labor income frontier is the simplest and the most compelling rationale for a comprehensive income tax which treats capital and labor income flows alike. Our tax scheme adopts a constant rate of progressivity (CRP) as popularized by Bénabou (2002) and Heathcote et al. (2017). The later paper shows that the CRP tax scheme approximates the actual tax and transfer system of the U.S. economy pretty well. The CRP tax scheme is often labeled the HSV tax scheme in the literature.

Saez (2001) derived an optimal top tax rate formula for high income earners, finding that the optimal top tax rate is an increasing function of ratio $y_m/\bar{y}$ (y_m denotes the mean of income above the threshold $\bar{y}$). He showed that the ratio $y_m/\bar{y}$ remains constant at the high end of the empirical earnings distributions and concluded that they are exactly Pareto distributions with $y_m/\bar{y} = a/(a-1)$, where $a > 1$ is the Pareto parameter.

Will the Pareto parameter a depend on the top tax rate imposed? Saez (2001) showed that it will not in the context of the optimal taxation model *à la* Mirrlees (1971). Based on his other study, he also stated (p. 212): "Empirically, in the U.S. a does not seem vary systematically with the level of the top rate." In this paper we extend Saez's finding to the degree of tax progressivity. Lee et al. (2023) found that higher capital income tax rates are associated with higher wealth Pareto exponents.

[1] During the past two decades, the Bewley-Huggett-Aiyagari model has become a workhorse for policy evaluations in the current state-of-the-art macroeconomics that jointly addresses aggregate and inequality issues. For surveys of the literature, see Heathcote et al. (2009), Guvenen (2011), Quadrini and Ríos-Rull (2015), Krueger et al. (2016) and Chapter 18 of Ljungqvist and Sargent (2018).

[2] This feature approximately holds in the real world. For example, the current U.S. personal income tax code is imposed on the sum of different types of personal incomes and does not distinguish between them in general (say, wages and salaries vs. interest and dividends) when computing tax liabilities. A notable exception with discrimination between labor and capital income is the so-called "dual income tax" (see Sørensen (1994) for details).

2 Model

Our model builds on Achdou et al. (2022), which is a continuous-time version of the Bewley-Huggett-Aiyagari model. Following Angeletos (2007) and Panousi (2012), we introduce idiosyncratic investment risk into the model via the private firms.[3]

Time is continuous, indexed by $t \in [0, \infty)$. The economy is populated by a continuum of infinitely-lived households of unit mass. The household is indexed by $i \in [0, 1]$. Each household consists of a worker and a producer ("entrepreneur"). The worker supplies one unit of labor inelastically to the labor market. The entrepreneur runs a privately-held firm ("family business") by hiring labor from the labor market and accumulating capital within his own family business. Each household is atomistic and thus a price taker.

The evolution of capital k is given by the household budget,

$$dk(t) = d\pi(t) + [w(t)z(t) - c(t) - \delta k(t) - T(y(t))]\, dt, \tag{1}$$

where $d\pi(t)$ is the profit earned from running its own family business, $w(t)$ is the wage rate, $z(t)$ is the labor productivity shock, $c(t)$ is the household's consumption, δ is the depreciation rate of capital $k(t)$, and $T(y(t))$ is the income tax scheme imposed by the government. We let $z(t)$ follow a two-state Poisson process $z(t) \in \{z_1, z_2\}$ with $z_2 > z_1 > 0$. The process jumps from state 1 to state 2 with intensity λ_1 and vice versa with intensity λ_2. The two-state process is a simplification. There could be arbitrarily many finite states instead of two.

We assume that the household can only participate in a risky business and cannot access the risk-free asset (like government bonds). Our main result hinges upon this important assumption. If there are bonds, say in zero net supply, the interest rate will be endogenously determined through the market clearing condition. Then the tax progressivity will affect the Pareto exponent, but only through the general equilibrium effect.

Following Angeletos (2007) and Angeletos and Panousi (2009), we let

$$d\pi(t) = \left[Ak(t)^{\alpha} n(t)^{1-\alpha} - w(t)n(t)\right] dt + \sigma k(t) dB(t), \tag{2}$$

where $\alpha \in (0, 1)$, A represents the total factor productivity (TFP), $n(t)$ is the amount of labor hired by the family business, and $B(t)$ is a standard Brownian motion. The scalar σ

[3] In their online appendix (section G.4), Achdou et al. (2022) explicitly suggested that one particularly appealing interpretation of the risky asset is that the return of the risky asset is the return from owning and running a private firm as in Angeletos (2007).

measures the undiversified idiosyncratic investment risk due to market incompleteness.

The private firms have a Cobb-Douglas production function and they hire labor in a competitive market. Entrepreneurs choose $n(t)$ to achieve

$$\max_{n(t)} Ak(t)^{\alpha} n(t)^{1-\alpha} - w(t)n(t),$$

which yields the optimal labor hiring,

$$n(t) = \left(\frac{(1-\alpha)A}{w(t)} \right)^{\frac{1}{\alpha}} k(t). \tag{3}$$

Substituting the optimal $n(t)$ into (2) we have

$$d\pi(t) = r(t)k(t)dt + \sigma k(t)dB(t), \tag{4}$$

where $r(t)$ represents the mean rate of return on capital with $r(t) = \alpha A^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w(t)} \right)^{\frac{1}{\alpha}-1}$. The total labor supply in the economy is exogenous and constant. Thus, the aggregate production in the stationary economy is constant.

The income tax scheme $T(\cdot)$ is the same as in Bénabou (2002) and Heathcote et al. (2017),

$$T(y) = y - \varphi y^{1-\zeta},\ 0 < \zeta < 1, \tag{5}$$

where $\varphi > 0$ and the parameter ζ determines the progressivity of income taxation. Since $\zeta = \frac{yT''(y)}{1-T'(y)}$ for all $y > 0$, the tax scheme is known as the CRP tax scheme and it is progressive, proportional, and regressive, depending on whether $\zeta > 0$, $\zeta = 0$, or $\zeta < 0$. We focus on $\zeta > 0$ in this paper. $T(y(t))$ is imposed on the household's total income $y(t) = r(t)k(t) + w(t)z(t)$, which is in line with the tax code in the U.S. Because $T(y(t))$ could be negative if $y(t)$ is low, as noted by Heathcote et al. (2017), the tax function (5) is best seen as a tax and transfer scheme.[4]

The government is required to balance its budget in each period,

$$g \int_0^1 y^i di = \int_0^1 T\left(y^i\right) di, \tag{6}$$

[4]If the tax function is $T(y) = \tau(y - \varphi y^{1-\zeta})$ with $\tau \in (0,1)$ instead, then the after-tax capital income will not be negligible at the top, and hence the Pareto exponent will again depend on the tax progressivity through the general equilibrium effect.

where g denotes the fraction of the aggregate output $Y(t) = \int_0^1 y^i(t)di$ that is devoted to the government consumption. In the numerical analysis, we let the government choose the pair (g, ζ), with φ being determined residually by the balanced budget (6). As in Judd (1985), Conesa et al. (2009), Hsu and Yang (2013), and Chang and Park (2021), we abstract from the issue of government bonds.

The household suffers from idiosyncratic labor income risk and idiosyncratic investment risk. Both risks give rise to the precautionary saving motive. The household's utility maximization problem is

$$\max_{\{c(t),k(t)\}_{t=0}^{\infty}} \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} u(c(t))dt,$$

where $\rho > 0$ is the time discount rate, $\mathbb{E}_0$ is the expectation operator conditional on the information set at time 0, and $u(c(t))$ is the instantaneous utility function. The household is subject to the constraint,

$$dk(t) = \left[\varphi(r(t)k(t) + w(t)z(t))^{1-\zeta} - \delta k(t) - c(t)\right] dt + \sigma k(t)dB(t), \tag{7}$$

plus $k(t) \geq 0$ for all $t \geq 0$. Equation (7) is obtained by combining equations (1), (4), and (5).

Let $v(k(t))$ be the value function of the household's problem,

$$v(k(t)) = \max_{\{c(\tau),k(\tau)\}_{\tau=t}^{\infty}} \mathbb{E}_t \int_t^{\infty} e^{-\rho(\tau-t)} u(c(\tau))d\tau,$$

where $\mathbb{E}_t$ is the expectation operator conditional on the information set at time t. The valuation function $v(k)$ satisfies the Hamilton-Jacobi-Bellman (HJB) equation,

$$\rho v_j(k) = \max_c u(c_j) + v_j'(k)s_j(k) + \frac{1}{2}v_j''(k)\sigma^2 k^2 + \lambda_j\left(v_{-j}(k) - v_j(k)\right), \; j = 1, 2, \tag{8}$$

where $s_j(k) = \varphi(rk + wz_j)^{1-\zeta} - \delta k - c_j(k)$. We adopt the convention that $-j = 2$ when $j = 1$, and $-j = 1$ when $j = 2$. We can obtain the consumption policy function $c_j(k)$ and saving policy function $s_j(k)$ by solving the HJB equation. The first-order condition is given by

$$v_j'(k) = u'(c_j(k)), \; j = 1, 2, \tag{9}$$

which gives $c_j(k) = (u')^{-1}(v_j'(k))$.

The cross-section capital distributions $f_j(k, t)$, $j = 1, 2$, are governed by the Kolmogorov

Forward (KF) equation,

$$\frac{\partial}{\partial t} f_j(k,t) = \frac{1}{2}\frac{\partial^2}{\partial k^2}\left[\sigma^2 k^2 f_j(k,t)\right] - \frac{\partial}{\partial k}\left[s_j(k) f_j(k,t)\right] - \lambda_j f_j(k,t) + \lambda_{-j} f_{-j}(k,t),\ \ j=1,2. \quad (10)$$

Letting $\frac{\partial}{\partial t} f_j(k,t) = 0$ in equation (10), the stationary distributions $f_j(k)$, $j=1,2$, satisfy

$$0 = \frac{1}{2}\frac{d^2}{dk^2}\left[\sigma^2 k^2 f_j(k)\right] - \frac{d}{dk}\left[s_j(k) f_j(k)\right] - \lambda_j f_j(k) + \lambda_{-j} f_{-j}(k),\ \ j=1,2. \quad (11)$$

We solve the consumption function from the HJB equation and then obtain the saving function. From the KF equation, we solve the wealth distribution function, which is the key element of aggregation in the model. The saving function is the only channel through which the individual behavior influences the aggregate variables of the economy in our model.

The competitive equilibrium of the economy is standard.

Definition 1 *A stationary competitive equilibrium is defined as a pair of prices (r, w), government policy $T(y)$, and individual policy functions $c(k)$, $s(k)$, and $n(k)$, such that the following conditions hold:*

(i) given (r, w) and $T(y)$, the plans $c(k)$, $s(k)$, and $n(k)$ are optimal for the household;

(ii) the government budget is balanced as in equation (6);

(iii) the stationary distribution $f_j(k)$ for $j = 1, 2$ satisfies equation (11);

(iv) the labor market clears:

$$\sum_{j\in\{1,2\}} \int_0^\infty n(k) f_j(k) dk = N(t) \equiv \frac{z_1\lambda_2 + z_2\lambda_1}{\lambda_1 + \lambda_2}.$$

The wealth distribution $f_j(k)$ influences the aggregate economy through the labor market equilibrium. Even though labor supply is exogenously given, labor demand is determined by the wealth distribution. The wage rate is determined by the labor market equilibrium. On the other hand, the equilibrium wage rate and return on capital influence the saving function of households. The stationary wealth distribution is derived from the KF equation, which is determined by the saving function. Thus, $f_j(k)$ and the aggregate economy interact. For $\zeta > 0$, we always find a stationary competitive equilibrium in our numerical study.

3 Theoretical results

This section establishes our theoretical results, which extend some theoretical results in Achdou et al. (2022) to our economy with the imposition of the CRP income tax scheme.

3.1 Consumption and saving behavior of the wealthy

We focus on a commonly used utility function.

Assumption 1 $u(c) = \frac{c^{1-\eta}}{1-\eta},\ \eta > 0.$

For any two functions f and g, we let $f \sim g$ denote $\lim_{k\to\infty} f(k)/g(k) = 1$, namely, f behaves like g as $k \to \infty$. We have some useful results characterizing the asymptotic property of consumption and saving policy functions under different progressivities of income taxation.

Lemma 1 *Impose Assumption 1 and the tax scheme (5) with $\zeta > 0$. As $k \to \infty$, the consumption policy function $c_j(k)$ and saving policy function $s_j(k)$, $j = 1, 2$, have the following asymptotic properties:*

$$c_j(k) \sim \left[\frac{\rho + (1-\eta)\delta}{\eta} + \frac{\sigma^2(1-\eta)}{2}\right] k, \quad s_j(k) \sim -\left[\frac{\rho+\delta}{\eta} + \frac{\sigma^2(1-\eta)}{2}\right] k.$$

A heuristic proof of Lemma 1 is in Online Appendix.[5]

Several features of Lemma 1 stand out. First, the consumption and saving functions of the very wealthy do not depend on their labor income wz_j, regardless of the value of $\zeta > 0$. The result arises because the ratio wz_j/k becomes infinitesimal as k approaches infinity. Second, the consumption and saving functions are asymptotically linear in k for $\zeta > 0$. Thus, $\lim_{k\to\infty} \frac{s_j(k)}{k}$ exists and does not depend on j. Third, the consumption and saving behavior of the very wealthy does not respond to the degree of tax progressivity. This result is intuitive, in that the after-tax income, $\varphi(rk + wz_j)^{1-\zeta}$, is an infinitesimal fraction of very high k as $\zeta > 0$.

[5]It should be noted that Lemma 1 is not really a proposition in the mathematical sense, as the order of magnitude argument is heuristic. In discrete time, Ma and Toda (2021, 2022) rigorously showed the asymptotic linearity of consumption functions with Markovian shocks. See also Gouin-Bonenfant and Toda (2023).

3.2 The Pareto tail

The following result characterizes the upper tail of the stationary wealth distribution.

Proposition 1 *Impose Assumption 1 and the tax scheme (5) with $\zeta > 0$. Letting $\eta(\eta-1)\sigma^2 < 2(\rho+\delta)$, there exists a unique stationary wealth distribution which follows an asymptotic power law, i.e. $1 - F(k) \sim \kappa k^{-a}$ (κ is a constant) as $k \to \infty$, with*

$$a = 2 - \eta + \frac{2(\rho + \delta)}{\eta\sigma^2}.$$

Proof: See Online Appendix.

The above result extends the result of the fat-tailed wealth distribution in Benhabib et al. (2015) to a general equilibrium setting with the incorporation of the labor market and with the imposition of the CRP income tax scheme. Proposition 1 explicitly gives the expression of the Pareto exponent.[6]

For $\zeta > 0$, $s_j(k)$ is asymptotically linear in k. To obtain the stationary wealth distribution, the precautionary saving caused by risk can not be too large. Specifically, we need a technical condition: $\eta(\eta - 1)\sigma^2 < 2(\rho + \delta)$.

The Pareto exponent of the wealth distribution does not depend on tax progressivity when $\zeta > 0$. This result reflects the fact that the Pareto exponent is determined by the linear component of the policy function of the wealthy household. As shown in Lemma 1, the slope of the saving function does not depend on tax progressivity as $\zeta > 0$.

Since $y = rk + wz$ and z is bounded, the Pareto exponent of the income distribution is the same as that of the wealth distribution. Thus, we have

Corollary 1 *Impose Assumption 1 and the tax scheme (5) with $\zeta > 0$. The upper tail of the income distribution follows an asymptotic power law, i.e. $1 - F(y) \sim \varsigma y^{-a}$ (ς is a constant) as $y \to \infty$, with*

$$a = 2 - \eta + \frac{2(\rho + \delta)}{\eta\sigma^2}.$$

As in Proposition 1, the Pareto exponent a is independent of tax progressivity $\zeta > 0$.

[6] Blanchet et al. (2022) showed that the Pareto exponent is an important index for estimating top income shares by using tax data.

4 Numerical results

This section numerically verifies our theoretical result (Corollary 1): a higher or lower progressivity of income taxation does not alter the fatness of the Pareto upper tail of income distributions.

Heathcote et al. (2017) estimated the CRP tax scheme (5) for the U.S. economy, finding $\zeta = 0.181$.[7] As noted in the Introduction, they showed that the CRP tax scheme approximates the actual tax and transfer system of the U.S. economy pretty well. We adopt their estimation as our benchmark. Table 1 reports the parameter values of the model used in our numerical analysis. As for the details of parameterization, we refer to Yang et al. (2023).

Table 1: Parameters

Coefficient of relative risk aversion	$\eta = 1.1$
Time discount rate	$\rho = 0.04$
Depreciation rate	$\delta = 0.03$
Capital income share	$\alpha = 1/3$
Progressivity of income taxation	$\zeta = 0.181$
Government purchases to GDP ratio	$g = 0.189$
Volatility of Brownian motion	$\sigma = 0.45$
Probability of transition for earnings	$\{0.047, 0.5\}$
Labor productivities	$\{0.15, 3.5\}$
Total factor productivity	$A = 0.6$

Following Saez (2001), we let $y_m = \int_{\bar{y}}^{\infty} y f(y) dy / \int_{\bar{y}}^{\infty} f(y) dy$, that is, y_m denotes the mean of income above threshold $\bar{y}$. The ratio $y_m/\bar{y}$ is supposed to approach $\frac{a}{a-1}$ as $\bar{y}$ approaches infinity. Figure 1 reports $y_m/\bar{y}$ in our model under $\zeta = 0.181$.[8]

[7] To estimate the parameters of the CRP tax scheme, Heathcote et al. (2017) use the definition of income including both labor earnings and capital income ("labor earnings, self-employment income, private transfers (alimony, child support, help from relatives, miscellaneous transfers, private retirement income, annuities, and other retirement income), plus income from interest, dividends, and rents." (p. 1699)). Our setup for income taxation is consistent with the definition of income in their estimation.

[8] The household with income greater than 10 accounts for 0.07% of the total population in the model.

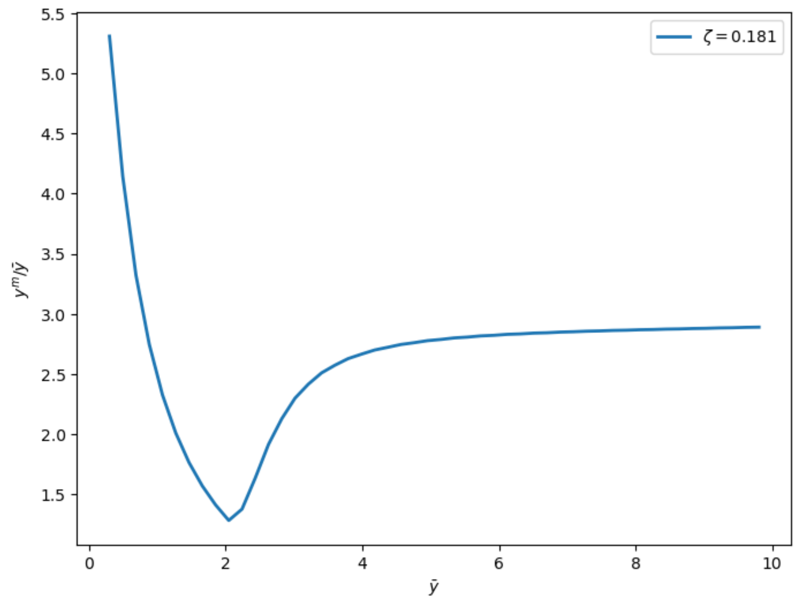

Figure 1: Ratio mean income above $\bar{y}$ divided by $\bar{y}$, $y_m/\bar{y}$

Diamond and Saez (2011, Figure 2) showed that the value of a is extremely stable in the United States for $\bar{y}$ above \$300,000 and it equals approximately 1.5. Using the formula in Corollary 1, we find $a = 1.53$, which implies the ratio $y_m/\bar{y}$ converges toward a value of around 2.9. Our value of $a = 1.53$ under the tax progressivity $\zeta = 0.181$ is close to $a = 1.5$ reported in Diamond and Saez (2011).

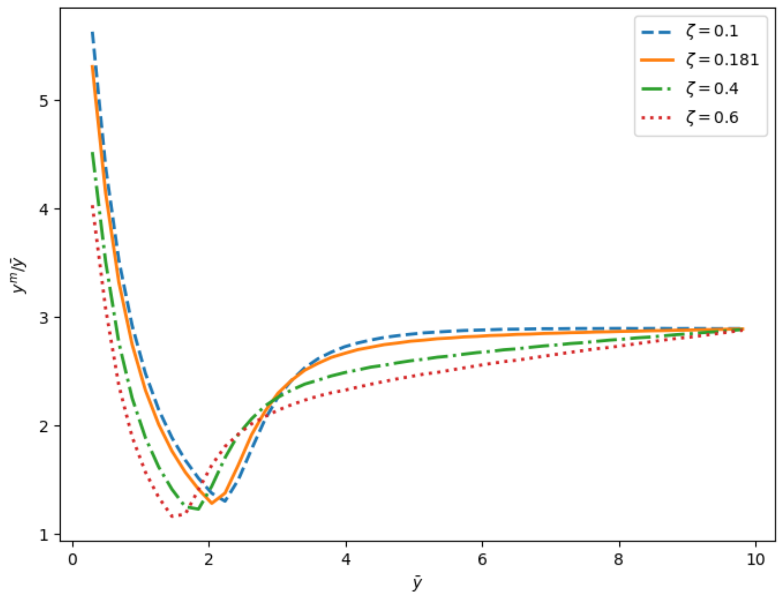

Figure 2: $y_m/\bar{y}$ under different tax progressivities

In Figure 2, we report the pattern of $y_m/\bar{y}$ under different tax progressivities, including $\zeta = 0.1$, 0.4, and 0.6, along with $\zeta = 0.181$ (the U.S. data).[9] A salient feature of the figure is that, despite the values of ζ vary significantly, all of $y_m/\bar{y}$ under different ζ converge toward the one under $\zeta = 0.181$. That is, all of $y_m/\bar{y}$ under different ζ converge toward the value around 2.9. This feature numerically verifies the theoretical result of Corollary 1 that varying tax progressivity does not affect the Pareto exponent of income distributions.

5 Conclusion

In a continuous-time version of the Bewley-Huggett-Aiyagari model, this paper shows theoretically and numerically that a higher or lower progressivity of income taxation does not alter the fatness of the Pareto upper tail of income distributions. The policy implications of this result is surely worthy of study in the future.

[9]We provide the details of producing Figure 2 in Online Appendix **??**.

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