# Bequests, Estate Taxes, and Wealth Distributions*

Jing Wan
Tianjin University

Shenghao Zhu
National University of Singapore

June 3, 2017

**Abstract**

Bossmann et al. (2007) ("Bequests, taxation and the distribution of wealth in a general equilibrium model," *Journal of Public Economics*, 91, 1247-1271), find that estate taxes reduce the long-run wealth inequality. This result contrasts with the findings of previous literatures with idiosyncratic labor efficiency risk. We use a decomposition technique, developed by Davies (1986) ("Does redistribution reduce inequality?" *Journal of Labor Economics*, 4, 538-559), to reinvestigate the impact of estate taxes on the long-run wealth inequality. We find that the different results of estate taxes are due to the different redistribution effects.

*A previous version of this paper was circulated as "Intergenerational links, taxation, and wealth distributions." We thank Daniel Barczyk, Jess Benhabib, Alberto Bisin, Yongheng Deng, Aditya Goenka, Tomoo Kikuchi, Christian Kleiber, Haoming Liu, Baochun Peng, Ariell Reshef, Thomas Sargent, Klaus Wälde, C.C. Yang, Ting Zeng, and Jie Zhang. Shenghao Zhu acknowledges the financial support from the Start-up Grant (R-122-000-138-133) of National University of Singapore.

*JEL classification:* D31; E21; H23

*Keywords:* Wealth inequality; Bequest motives; Estate taxes; Lorenz dominance

# 1 Introduction

Bossmann et al. (2007) find that estate taxes reduce the long-run wealth inequality. This result contrasts with the findings of previous literatures with idiosyncratic labor efficiency risk, such as Becker and Tomes (1979) and Davies (1986). These papers show that estate taxes usually increase the long-run wealth inequality.[1] In this paper we use the decomposition technique developed by Davies (1986) to reinvestigate the impact of estate taxes on the long-run wealth inequality. We find that the redistribution effect plays an important role in determining the effect of the estate tax on the long-run wealth inequality.

Following Bossmann et al. (2007), we investigate the impact of bequest motives and estate taxes on wealth inequality in a two-period overlapping generations (OLG) heterogeneous-agents model. Each agent lives for two periods: the young period and the old period. There are a continuum of measure 1 families in the economy. Each family consists of one parent and one child. Each young agent supplies 1 unit of labor inelastically and has idiosyncratic labor efficiency risk. Old agents do not have labor earnings. Agents have "joy of giving" bequest motives. The government collects the estate-tax revenue and gives a lump-sum transfer to the young generation. The government has a balanced budget in every period.

[1] Simulations of Davies and Kuhn (1991) show that estate taxes reduce wealth inequality in the short run, even though they increase inequality in the long run.

Becker and Tomes (1979) find that a progressive tax-subsidy system tends to increase the long-run inequality. Davies (1986) develops a decomposition technique to study the impact of the estate tax on the long-run wealth inequality in the Becker-Tomes model. He separates the inheritance effect of the estate tax from the redistribution effect. And both of these two effects increase the long-run wealth inequality. Thus estate taxes increase the long-run wealth inequality in the Becker-Tomes model.

As Bossmann et al. (2007), we find that estate taxes reduce the long-run wealth inequality. We use the decomposition technique, developed by Davies (1986), to reinvestigate the impact of estate taxes on the long-run wealth inequality. In our model the inheritance of bequests decreases the long-run wealth inequality through averaging labor efficiency luck in a lineage. The inheritance effect of the estate tax increases the long-run inequality through interfering with the inheritance of bequests. In this respect, our model and Bossmann et al. (2007) are in line with previous literatures with idiosyncratic labor efficiency risk, such as Becker and Tomes (1979) and Davies (1986).

We find that it is the redistribution effect of the estate tax that causes different effects on the long-run wealth inequality. The redistribution effect is the effect of the lump-sum transfer on wealth inequality. The increase of the transfer reduces wealth inequality. Agents in our model have "joy of giving" bequest motives. Raising estate taxes increases government revenues and subsidies. The redistribution effect decreases wealth inequality in our model and Bossmann et al. (2007).

In our model the inheritance effect of the estate tax and the redistribution effect work in opposite directions. The redistribution effect dominates the inheritance effect. The net effect of the estate tax is to reduce wealth inequality. These findings help us to understand the impact of estate taxes on the long-run wealth inequality. Davies (1986) finds that the inheritance effect of the estate tax increases the long-run wealth inequality in the model with idiosyncratic labor efficiency risk. This result holds for both altruistic bequest motives and "joy of giving" bequest motives, as long as the wealth accumulation equation is linear. Thus the redistribution effect plays an important role in determining the net effect of the estate tax on wealth inequality. If the redistribution effect increases wealth inequality, as in Becker and Tomes (1979) and Davies (1986), then the net effect of the estate tax increases the long-run wealth inequality. If the redistribution effect decreases wealth inequality and it dominates the inheritance effect, as in our model and Bossmann et al. (2007), then the net effect of the estate tax reduces the long-run wealth inequality.

We also extend our benchmark model in two directions. In the first extension, we include housing as a new asset in the model. In this extension we show that all the theoretical results of the long-run wealth inequality in the benchmark model still hold. In the other extension, we permit the agent to live for more than two periods. In this extension we use a calibration exercise to illustrate that the results of the long-run wealth inequality in our benchmark model are still true. Estate taxes reduce the long-run wealth inequality.

Our findings also help us to understand how different ways of modelling bequest motives influence the impact of estate taxes on wealth inequality. Different forms of bequest motives, altruism and "joy of giving", do not influence the inheritance effect of the estate tax, but they imply different redistribution effects of the estate tax. In a model with "joy of giving" bequest motives the government can collect more tax revenues when it increases the estate tax. However, in a model with altruistic bequest motives the government collects less tax revenues when it increases the estate tax. Since a higher lump-sum transfer always reduces the wealth inequality, government revenues influence the long-run wealth distribution. Thus different forms of bequest motives influence the impact of estate taxes on wealth inequality through the redistribution effect.

Although previous literatures, such as Gale and Perozek (2001), find that different forms of bequest motives influence the impact of estate taxes on wealth accumulation, few papers investigate how different forms of bequest motives influence the impact of estate taxes on wealth inequality. Our paper fills this gap. This research is important given that empirical researches have not found evidences to distinguish these two bequest motives: altruism and "joy of giving". In a recent literature review, Kopzuk (2013) states that "Bequest motives are the key building block for theoretical analysis of taxation of transfers, but the empirical literature has not settled on a clear answer to the question about the nature of bequest motivations" (Page 331 of Kopzuk (2013)).

We do not incorporate precautionary savings motives into our model and agents have linear policy functions.[2] The linear property permits us to use Lorenz dominance to investigate the impact of bequest motives and

[2]Literatures of incomplete-market heterogeneous-agents models, such as Aiyagari (1994), Castaneda et al. (2003) and De Nardi (2004), Benhabib et al. (2015), and De Nardi and Yang (2016), incorporate precautionary savings motives into their models. They solve agent's policy functions numerically and simulate the stationary wealth distribution. Benhabib et al. (2011) also find agent's policy functions explicitly. They use idiosyncratic investment risk to generate the observed fat tail of the wealth distribution in the United States.

estate taxes on wealth inequality. Lorenz dominance is widely used in literatures of income and wealth inequality. For example, Chatterjee (1994) uses Lorenz dominance to discuss wealth distribution in a neoclassical growth model. Zilcha (2003) uses Lorenz dominance to study the income distribution in an economy with two types of intergenerational transfers: investment of parents in the education of their offspring, and capital transfer. Early literatures include, among others, Atkinson (1970) and Rothschild and Stiglitz (1973). For a recent review on this topic see Gajdos and Weymark (2012).

The rest of this paper is organized as follows. Section 2 presents the basic structure of our model. Section 3 discusses the stationary wealth distribution. In Section 4 we introduce housing into our benchmark model. We extend the benchmark model to a life-cycle model in Section 5. Section 6 concludes the paper. Appendix contains most of the proofs.

# 2 The model

Our model is an overlapping generations heterogeneous-agents economy. There are a continuum of measure 1 families in the economy. Each family consists of one parent and one child. Each agent lives for two periods: the young period and the old period. Each old agent gives birth to one child. The population of the economy keeps constant.

## 2.1 The agent's problem

Young agents work and earn labor earnings. Each young agent supplies 1 unit of labor inelastically. But young agents have idiosyncratic labor efficiency risk $l_t$. We assume

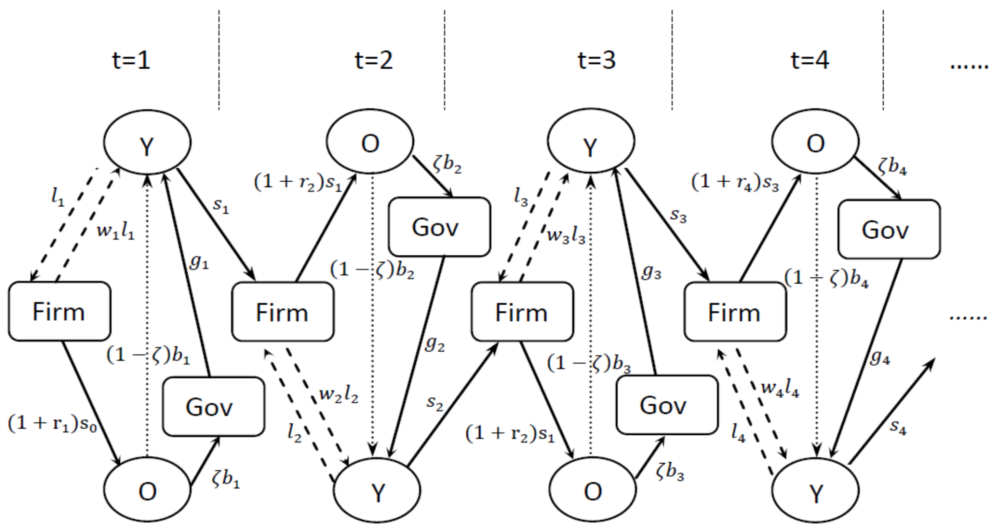
*Assumption 1:* $\{l_t\}$ is stationary and ergodic.[3]

*Assumption 2:* $l_t > 0$ has a finite mean.[4] Without loss of generality,

$$E(l_t) = 1.$$

The wage rate per efficiency unit is $w_t$. An young agent born at period $t$ consumes $c_t^y$ in the first period of his life. $s_t$ denotes his savings. The interest rate in period $t+1$ is $r_{t+1}$. An old agent does not have labor income. His consumption is $c_{t+1}^o$. He leaves the bequest $b_{t+1}$ to his child in the second period of his life. The government collects the estate tax $\zeta b_{t+1}$, where $\zeta \in [0, 1)$ is the estate tax rate. In period $t$, an young agent receives bequests $(1-\zeta)\, b_t$. He draws his labor efficiency $l_t$ and receives the lump-sum transfer $g_t$ from the government.

Figure 1 shows the timing of the model.




---

[3] We use $\{x_t\}$ to represent a sequence in this paper.

[4] Note that we do not need to assume that $Var(l_t) < \infty$.

Figure 1: The timing of the model

The young agent first draws his labor efficiency $l_t$ and then the agent makes consumption and savings decisions. Thus the agent's problem is a deterministic optimization problem. The old agent has a "joy of giving" bequest motive. Both the utility functions of the consumption and the bequest have the form of constant relative risk aversion (CRRA). The young agent's problem is

$$\max_{c_t^y, s_t, c_{t+1}^o, b_{t+1}} \frac{(c_t^y)^{1-\eta}-1}{1-\eta} + \beta \left[ \frac{(c_{t+1}^o)^{1-\eta}-1}{1-\eta} + \chi \frac{\left[(1-\zeta)\, b_{t+1}\right]^{1-\eta}-1}{1-\eta} \right] \tag{1}$$

$$\begin{aligned} s.t. \quad c_t^y + s_t &= w_t l_t + (1-\zeta)\, b_t + g_t, \\ c_{t+1}^o + b_{t+1} &= (1+r_{t+1}) s_t, \end{aligned}$$

where $\eta \geq 1$ is the coefficient of relative risk aversion, $\beta \in (0,1)$ is the time discount factor, and $\chi > 0$ represents the bequest motive intensity.

The agent's optimal policy functions are

$$c_{t+1}^o = \frac{1}{1+\chi^{\frac{1}{\eta}}(1-\zeta)^{\frac{1-\eta}{\eta}}}(1+r_{t+1})s_t,$$

$$b_{t+1} = \frac{1}{1+\chi^{-\frac{1}{\eta}}(1-\zeta)^{\frac{\eta-1}{\eta}}}(1+r_{t+1})s_t,$$

$$c_t^y = \frac{1}{1+\tilde{\beta}_{t+1}^{\frac{1}{\eta}}}\left[w_t l_t + (1-\zeta)\, b_t + g_t\right],$$

and

$$s_t = \frac{1}{1+\tilde{\beta}_{t+1}^{-\frac{1}{\eta}}}\left[w_t l_t + (1-\zeta)\, b_t + g_t\right],$$

where $\tilde{\beta}_{t+1} = \beta \left[1 + \chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}}\right]^{\eta} (1+r_{t+1})^{1-\eta}$.

From optimal policy functions of $b_{t+1}$ and $s_t$, we derive the agent's wealth accumulation equation,

$$s_t = \frac{1}{1 + \tilde{\beta}_{t+1}^{-\frac{1}{\eta}}} \left[ w_t l_t + (1-\zeta)\, \varphi (1+r_t) s_{t-1} + g_t \right], \tag{2}$$

where $\varphi = \frac{1}{1+\chi^{-\frac{1}{\eta}} (1-\zeta)^{\frac{\eta-1}{\eta}}}$.

These linear functions, induced by the CRRA utility functions, bring us conveniences to describe both the aggregate economy and the stationary wealth distribution. Under the linear wealth accumulation equation, the aggregate economy only depends on the mean of the wealth accumulation. Other moments of the wealth distribution do not influence the aggregate economy. Thus we can use a nonlinear equation to describe the aggregate capital accumulation without characterizing the wealth distribution along the transition of the aggregate economy.

When we investigate the stationary wealth distribution, the linear wealth accumulation equation permits us to find an expression of the stationary wealth distribution. We also use the linear wealth accumulation equation to establish the Lorenz dominance relationship when we study the comparative statics of the stationary wealth distribution.

## 2.2 The firm's problem

A firm has the aggregate production in the economy,

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha},$$

where $A$ is the technology level, $Y_t$ is the output, $K_t$ is the capital, $L_t$ is the labor, and $\alpha$ is capital's share of income. The firm chooses $K_t$ and $L_t$ to maximize its profits,

$$\max_{K_t, L_t} \{AK_t^{\alpha} L_t^{1-\alpha} - w_t L_t - (r_t + \delta) K_t\},$$

where $\delta$ is the depreciation rate of capital.

The capital market and the labor market are competitive. Capital and labor are paid their marginal products. From the firm's problem we have

$$r_t = \alpha A K_t^{\alpha-1} L_t^{1-\alpha} - \delta, \tag{3}$$

and

$$w_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha}. \tag{4}$$

## 2.3 The government

The government collects the estate-tax revenue and gives a lump-sum transfer to the young generation. Each young agent receives the same subsidy $g_t$. The government has a balanced budget in every period. Thus we have

$$g_t = \zeta \int b_t di, \tag{5}$$

where $\int di$ denotes the aggregation of young agents.

## 2.4 The general equilibrium

The aggregate population of young agents, who are the workers in the economy, is 1. And we have $E(l_t) = 1$ from Assumption 2. Thus the labor-market clearing condition is

$$L_t = \int l_t di = 1, \tag{6}$$

where $\int di$ denotes the aggregation of young population.

The capital-market clearing condition is

$$K_{t+1} = \int s_t di, \tag{7}$$

where $\int di$ denotes the aggregation of young agents.

Aggregating equation (2) across young agents and using equations (6) and (7), and the government budget constraint (5), we have

$$K_{t+1} = \frac{1}{1 + \tilde{\beta}_{t+1}^{-\frac{1}{\eta}}} \left[ w_t + \varphi(1 + r_t)K_t \right]. \tag{8}$$

With the labor-market clearing condition (6), equations (3) and (4) imply that

$$r_t = \alpha A K_t^{\alpha - 1} - \delta, \tag{9}$$

and

$$w_t = (1 - \alpha) A K_t^{\alpha}. \tag{10}$$

Plugging equations (9) and (10) we have

$$K_{t+1} = \frac{1}{1 + \tilde{\beta}_{t+1}^{-\frac{1}{\eta}}} \left[ (1 - \alpha + \varphi\alpha) A K_t^{\alpha} + \varphi(1 - \delta)K_t \right]. \tag{11}$$

This nonlinear equation describes the law of motion of the aggregate capital.

We will concentrate on the steady-state aggregate economy in which the aggregate capital $K$, the wage rate $w$, and the interest rate $r$ are constant.

**Proposition 1** *The economy has a unique aggregate steady state. An economy with a higher bequest motive* $\chi$ *has a higher steady-state aggregate capital* $K$*.*

The young agent born in period $t$ has two incentives for accumulating wealth. The first incentive is for his own consumption in the old period $c^o_{t+1}$. The second incentive is for the bequest left to his child $b_{t+1}$ . The higher the agent's bequest motive, the higher the agent's saving incentive for the bequest. Thus the wealth accumulation is higher. In one extreme case there is no bequest motive, i.e. $\chi = 0$. Proposition 1 implies that the steady-state aggregate economy with the bequest motive $\chi > 0$ has a higher aggregate wealth level than the economy without bequest motives.

# 3 The wealth distribution

In this section we investigate the stationary distribution of individual wealth accumulation process in the steady-state aggregate economy. Following Bossmann et al. (2007) we use $a_{t+1}$ to denote the individual wealth (before being paid interest) in period $t+1$. Thus $a_{t+1} = s_t$.

From government's budget constraint (5), we have

$$g_t = g = \zeta\varphi(1+r)K, \tag{12}$$

in the steady-state aggregate economy.

Plugging equation (12) into equation (2) we have the agent's wealth accumulation equation in the steady-state aggregate economy,

$$a_{t+1} = c_3 l_t + c_4 a_t + c_5, \tag{13}$$

where $c_3 = \frac{1}{1+\tilde{\beta}^{-\frac{1}{\eta}}} w$, $c_4 = \frac{(1-\zeta)\varphi(1+r)}{1+\tilde{\beta}^{-\frac{1}{\eta}}}$, and $c_5 = \frac{\zeta\varphi(1+r)}{1+\tilde{\beta}^{-\frac{1}{\eta}}} K$.

Equation (13) is the main equation of our paper. Our aim is to investigate the stationary distribution of process $\{a_t\}$ in the steady-state aggregate economy. We will study the stationary distribution of $\{a_t\}$ using the linear relationship of equation (13). We also use this linear relationship to compare the stationary wealth distribution of different economies in Sections 3.1 and 3.2.

To study the stationary distribution of process $\{a_t\}$ we first characterize the coefficient $c_4$ in the wealth accumulation equation (13).

**Proposition 2** $0 \leq c_4 < 1$.

Since Proposition 1 shows that aggregate capital $K$ in the steady-state aggregate economy is finite, we have an intuitive way to understand Proposition 2. And the aggregate savings equal $K$. Suppose that $c_4 \geq 1$, then $a_t \to \infty$ almost surely as $t \to \infty$ . Then we have $K = \infty$. Thus we must have $c_4 < 1$ in the steady-state aggregate economy.

**Proposition 3** *The unique stationary distribution of* $\{a_t\}$ *is*[5]

$$a_\infty =_{st} c_3 \sum_{s=0}^{\infty} c_4^s l_s + \frac{c_5}{1-c_4}. \tag{14}$$

[5] Here $=_{st}$ denotes equality in distribution.

*And $a_t$ converges to $a_\infty$ in distribution, denoted by $a_t \to_{st} a_\infty$, as $t$ approaches infinity.*

Proposition 3 shows that $a_t$ converges to $a_\infty$ in distribution as $t$ approaches infinity. We use this important property of convergence in distribution when we investigate the impacts of bequest motives and estate taxes on the stationary wealth distribution. In these analyses we first establish intuitions and obtain results in static situations. Then we extend these results to stationary wealth distributions by showing that they still hold when processes approach limiting distributions.

Bossmann et al. (2007) assume that $Var(l_t) < \infty$. However, as noted by Bossmann et al. (2007), "the assumption of a finite variance is not satisfied for all commonly used distributional assumptions for $l_t$." (Footnote 9 on page 1255 of Bossmann et al. (2007)) To derive Proposition 3 we do not assume that $Var(l_t) < \infty$.

## 3.1 Bequest motives and wealth inequality

In order to emphasize the impacts of bequest motives on wealth distribution, we set estate tax rate $\zeta = 0$, following Bossmann et al. (2007). Thus $c_5 = 0$. The agent's wealth accumulation equation (13) becomes

$$a_{t+1} = c_3 l_t + c_4 a_t.$$

Following Bossmann et al. (2007), we assume that there are two economies: economy $A$ and economy $B$. Agents in economy $A$ do not have bequest motives, i.e. $\chi = 0$. Agents in economy $B$ have bequest motives, i.e. $\chi > 0$ (B for bequests). Let $a_\infty^A$ be the stationary wealth distribution of economy

$A$, and $a^B_\infty$ be the stationary wealth distribution of economy $B$.

In economy $A$ there is no bequest motive and $c_4 = 0$. Thus we have

$$a_{t+1} = c_3 l_t,$$

which has the same Lorenz curve as $l_t$. Thus $a^A_\infty$ has the same Lorenz curve as $l_t$.

In economy $B$ there are bequest motives. By Proposition 2 we have $0 < c_4 < 1$. Plugging $c_5 = 0$ into equation (14) we have

$$\begin{aligned} a^B_\infty &= {}_{st} c_3 \sum_{s=0}^{\infty} c_4^s l_s \\ &= {}_{st} \frac{c_3}{1-c_4} \sum_{s=0}^{\infty} (1-c_4) c_4^s l_s. \end{aligned}$$

Since $\frac{c_3}{1-c_4}$ is a constant, $a^B_\infty$ has the same Lorez curve as $Z \equiv \sum_{s=0}^{+\infty}(1 - c_4)c_4^s l_s$. The random variable $Z$ is a weighted average of random variables, $l_0$, $l_1$, $l_2$, $\cdots$. It should be more equal than $l_t$.[6] Our analysis of the impacts of bequests on wealth distribution starts from this observation. We extend this intuition to the comparison between stationary wealth distributions $a^A_\infty$ and $a^B_\infty$.

Let $L_X(p)$ be the Lorenz curve of a non-negative random variable $X$ with a finite positive mean.[7] Using the Lorenz curve, we define the Lorenz ordering.[8]

**Definition 1** *For two random variables $X$ and $Y$, $X$ Lorenz dominates $Y$ if and only if $L_X(p) \geq L_Y(p)$, $p \in [0,1]$, denoted as $X \succeq_L Y$.*

[6] We state this intuition formally in Lemma 2 of Appendix A.4.

[7] For the mathematical definition of the Lorenz curve, $L_X(p)$, see Gastwirth (1971).

[8] For inequality measures, our main reference book is Shaked and Shanthikumar (2010). A good reference of Lorenz dominance is Arnold (1987).

Another commonly used inequality measure is the coefficient of variation ($CV$). For a random variable $X$ with a finite variance, $CV(X)$ is defined as

$$CV(X) = \frac{\sqrt{Var(X)}}{E(X)}.$$

If both $X$ and $Y$ have finite variances, then $X \succeq_L Y$ implies $CV(Y) \geq CV(X)$.[9] Thus Lorenz dominance implies the order of the coefficient of variation.

Comparing stationary wealth distributions $a_\infty^A$ and $a_\infty^B$, we have

**Theorem 1** *Under Assumptions 1 and 2,* $a_\infty^B \succeq_L a_\infty^A$.

Theorem 1 implies that, an economy in which agents have bequest motives, has a more equal wealth distribution than an economy in which agents do not have bequest motives. Theorem 1 is the same as Proposition 1 of Davies (1986). While Davies (1986) uses altruistic bequest motives as in Becker and Tomes (1979), our paper uses "joy of giving" bequest motives as in Bossmann et al. (2007).

Bossmann et al. (2007) also find that an economy with bequest motives has a stationary wealth distribution which is more equal than that of an economy without bequest motives. Our result extends that of Bossmann et al. (2007) in three respects. First, we only assume that $\{l_t\}$ is a stationary and ergodic. Bossmann et al. (2007) assume that $\{l_t\}$ either is independent and identically distributed (*i.i.d.*) or follows a linear mean-reverting process.[10] Secondly, we do not assume that $Var(l_t) < \infty$. Bossmann et al.

[9] See pages 68-69 of Marshall and Olkin (2007).

[10] The linear process is

$$l_{t+1} = \bar{l} + v(l_t - \bar{l}) + \varepsilon_{t+1}$$

where $\bar{l} = 1$ and $0 < v < 1$. $\{\varepsilon_t\}$ is *i.i.d.* with a zero mean, a finite variance, and

(2007) uses the coefficient of variation as their inequality measure. Thus they need $Var(l_t) < \infty$ to insure that the variance of the wealth distribution is finite. Our inequality measures are the Lorenz curve and the Gini coefficient, which only require the existence of the mean of the wealth distribution. Thus we do not need $Var(l_t) < \infty$. Finally, our result implies that of Bossmann et al. (2007). Bossmann et al. (2007) derive the coefficient of variation of wealth, their inequality measure, by calculating the mean and variance of the wealth distribution. They find that $CV(a_\infty^A) > CV(a_\infty^B)$ because the increase in mean wealth "overcompensates the increase in the variance of the wealth." (Page 1249 of Bossmann et al. (2007)) Thus the coefficient of variation falls.[11] If $Var(a_\infty^A) < \infty$ and $Var(a_\infty^B) < \infty$, our Theorem 1 implies that $CV(a_\infty^A) \geq CV(a_\infty^B)$.[12]

## 3.2 Estate taxes and wealth inequality

When investigating the impacts of estate taxes on wealth distribution, we concentrate on the logarithmic utility as in Bossmann et al. (2007).

*Assumption 3:* Utility functions are logarithmic.

Let $\eta = 1$. Then the CRRA utility of Section 2 reduces to the logarithmic utility. We can solve the aggregate capital of the steady-state aggregate economy,

$$\bar{K} = \left( \frac{1 - \alpha + \chi}{1 + \frac{1}{\beta} + \delta\chi} A \right)^{\frac{1}{1-\alpha}}.$$

a lower bound sufficient to keep $l_{t+1} > 0$. This process is used in Davies (1986) and Davies and Kuhn (1991). Solon (1992) and Zimmerman (1992) use different data sets in the United States to study the intergenerational mobility and find that the elasticity of child's earnings with respect to parent's earnings is about 0.4.

[11] See comments after Proposition 1 about the increase of mean wealth caused by bequest motives.

[12] There is one minor difference between our result and that of Bossmann et al. (2007). We can only show $CV(a_\infty^A) \geq CV(a_\infty^B)$ while they show $CV(a_\infty^A) > CV(a_\infty^B)$.

The estate tax does not affect aggregate capital. Thus it does not influence the interest rate and the wage rate since we have $\bar{r} = \alpha A \left(\bar{K}\right)^{\alpha-1} - \delta$ and $\bar{w} = (1-\alpha)A\left(\bar{K}\right)^{\alpha}$ in the equilibrium. The estate tax has no general equilibrium effect.

From equation (13) we know that the individual wealth accumulation equation is

$$a_{t+1} = c_6 l_t + c_7 \left[(1-\zeta)a_t + \zeta\bar{K}\right], \tag{15}$$

with $c_6 = \frac{1}{1+\frac{1}{\beta(1+\chi)}}\bar{w}$ and $c_7 = \frac{1}{\left(1+\frac{1}{\beta(1+\chi)}\right)\left(1+\frac{1}{\chi}\right)}(1+\bar{r})$. Both $c_6$ and $c_7$ do not depend on the estate tax $\zeta$.

We use Proposition 3 to represent the stationary wealth distribution as

$$a_\infty =_{st} \sum_{s=0}^{\infty} c_8^s \left(c_6 l_s + c_7\zeta\bar{K}\right), \tag{16}$$

where $c_8 = c_7(1-\zeta)$. Davies (1986) uses a decomposition technique to investigate channels through which the estate tax influence the long-run wealth inequality in Becker and Tomes (1979). I use this decomposition technique to analyze these two channels in our model. The channel through which the estate tax influences $c_8$ is called the lag structure effect. The channel through which the estate tax influences the term $c_6 l_s + c_7\zeta\bar{K}$ is called the transfer effect. Thus we separate the inheritance effect of the estate tax from the redistribution effect. The lag structure effect represents the inheritance effect. Also the government collects the estate-tax revenue and then gives a lump-sum transfer to the young generation. Thus the transfer effect represents the redistribution effect.

### 3.2.1 The inheritance effect of the estate tax

Theorem 1 shows that the inheritance of bequests reduces the long-run wealth inequality through intergenerational sharing of labor efficiency luck in a lineage. The higher the estate tax $\zeta$, the smaller $c_8$ of equation (16). The wealth distribution becomes less equal due to the inheritance effect of the estate tax.

Davies (1986) uses altruistic bequest motives to study the inheritance effect of the estate tax on the long-run wealth inequality in Becker and Tomes (1979). He finds that the inheritance effect of the estate tax increases inequality through interfering with the inheritance of bequests (Page 547 of Davies (1986)). He also points out that the inheritance effect of the estate tax holds for both altruistic bequest motives and "joy of giving" bequest motives, as long as the wealth accumulation equation is linear.

The estate tax in our model has the same inheritance effect as in the previous literatures with idiosyncratic labor efficiency risk. We summarize these discussions on the inheritance effect of the estate tax in Table 1.

Table 1: The inheritance effect of the estate tax

| Models | Bequest motives | The inheritance effect of the estate tax |
|---|---|---|
| This paper<br>Bossmann et al. (2007) | "joy of giving" | To increase the long- run wealth inequality |
| Becker and Tomes (1979)<br>Davies (1986) | Altruism | To increase the long- run wealth inequality |

#### 3.2.2 The redistribution effect

The higher the estate tax $\zeta$, the higher the term $\zeta\bar{K}$ in equation (16), which reflects the lump-sum transfer from the government. Thus the inequality of $c_6 l_s + c_7\zeta\bar{K}$ becomes lower.[13] The wealth distribution becomes more equal due to the redistribution effect of the estate tax.

In our model agents have "joy of giving" bequest motives and logarithmic utility functions. Estate taxes have no impact on the aggregate wealth. Thus raising estate taxes increases government revenues and subsidies. The redistribution effect of the estate tax decreases wealth inequality. This is different from the redistribution effect of Davies (1986). Davies (1986) uses altruistic bequest motives as in Becker and Tomes (1979) and finds that the redistribution effect of increasing estate taxes usually increases wealth inequality. Numerical results of Davies (1986) show that agent's optimal reactions cause the tax base to reduce by a percentage more that the increase of the estate tax. Thus the transfer decreases in the long run. Our model contrasts with these literatures in the redistribution channel of the estate tax.

Bossmann et al. (2007) find that the form of bequest motives plays a crucial role for the impact of the estate tax on the long-run wealth inequality. However, they do not separate the two channels of the inheritance effect and the redistribution effect. Thus they do not show that different forms of bequest motives, altruism and "joy of giving", imply different redistribution effects of the estate tax. Our paper contributes to the literature by

[13]This intuition comes from the mathematical result that $X + a$ Lorenz dominates $X + b$ for any non-negative random variable $X$ with a finite positive mean and $a > b > 0$ (See Theorem 3.A.25 of Shaked and Shanthikumar (2010)). Thus $X + a$ is more equal than $X + b$.

finding this different redistribution effect of the estate tax in a model with "joy of giving" bequest motives.

We summarize these discussions on the redistribution effect of the estate tax in Table 2.

Table 2: The redistribution effect of the estate tax

| Models | Bequest motives | The redistribution effect of the estate tax |
|---|---|---|
| This paper<br>Bossmann et al. (2007) | "joy of giving" | To reduce the long- run wealth inequality |
| Becker and Tomes (1979)<br>Davies (1986) | Altruism | To increase the long- run wealth inequality |

### 3.2.3 The net effect

The inheritance effect and the redistribution effect of the estate tax work in opposite directions in our model. We then investigate the net effect of the estate tax on the long-run wealth inequality.

We start from a static case.

**Lemma 1** *For a non-negative random variable $X$ with a positive finite mean, if $0 \leq \hat{\zeta} \leq \zeta < 1$, then $(1-\zeta)X + \zeta E(X) \succeq_L (1-\hat{\zeta})X + \hat{\zeta} E(X)$. Thus $(1-\zeta)X + \zeta E(X) \preceq_{cx} (1-\hat{\zeta})X + \hat{\zeta} E(X)$.*

A flat tax plus a lump-sum transfer is equivalent to a progressive tax since the effective average tax rate is increasing in wealth.[14] The higher the

[14] For an individual with before-tax wealth $x$, the effective average tax rate is

$$\frac{x - [(1-\zeta)\,x + \zeta E(X)]}{x} = \zeta \left[1 - \frac{E(X)}{x}\right],$$

which is increasing in $x$.

tax rate $\zeta$, the higher the lump-sum transfer. And the wealth distribution becomes more equal.[15]

To highlight the main finding of our decomposition results, we only concentrate on the case of *i.i.d.* $\{l_t\}$, when we study the net effect of the estate tax. Future work could investigate whether the Lorenz dominance result holds for the realistic case of $\{l_t\}$ which is correlated along generations.

*Assumption 4:* $\{l_t\}$ is *i.i.d.*

Let $a_\infty^\zeta$ be the stationary wealth distribution of an economy with an estate tax $\zeta$, and $a_\infty^{\hat{\zeta}}$ be the stationary wealth distribution of an economy with an estate tax $\hat{\zeta}$.

**Theorem 2** *Under Assumptions 1∼4, we have* $a_\infty^\zeta \succeq_L a_\infty^{\hat{\zeta}}$ *for* $\zeta \geq \hat{\zeta}$.

Theorem 2 extends the intuition in the static situation to stationary wealth distributions. For two economies with different estate tax rates, the economy with a higher estate tax rate has a more equal stationary wealth distribution. Bossmann et al. (2007) assume that $\{l_t\}$ either is *i.i.d.* or follows a linear mean-reverting process. Using the coefficient of variation as their inequality measure, Bossmann et al. (2007) show that estate taxes reduce wealth inequality.[16]

Following Davies (1986), we use a decomposition technique to investigate channels through which the estate tax influence the long-run wealth inequality. As in the previous literatures with idiosyncratic labor efficiency risk, the inheritance effect of the estate tax increases the long-run wealth inequality. More importantly, we find that the redistribution effect of the

[15] See Fellman (1976) for a study on the effect of progressive taxes on income distributions.

[16] In a simulation exercise Bossmann et al. (2007) find that the estate tax reduces the Gini coefficient of the long-run wealth distribution. Our theoretical result of Theorem 2 supports the simulation results of the Gini coefficient in Bossmann et al. (2007).

estate tax decreases wealth inequality in our model. Theorem 2 shows that the redistribution effect of raising estate taxes dominates the inheritance effect. And estate taxes reduce wealth inequality in our model.

In Becker and Tomes (1979) and Davies (1986), the inheritance effect and the redistribution effect of the estate tax increase the long-run wealth inequality. These two effects work in the same direction. Thus the net effect of increasing estate taxes on the long-run wealth distribution is disequalizing. We briefly review some main results of Becker-Tomes models in Appendix A.8.

We summarize these discussions on the net effect of the estate tax in Table 3.

Table 3: The net effect of the estate tax

| Models | Bequest motives | The net effect of the estate tax |
|---|---|---|
| This paper<br>Bossmann et al. (2007) | "joy of giving" | To reduce the long- run wealth inequality |
| Becker and Tomes (1979)<br>Davies (1986) | Altruism | To increase the long- run wealth inequality |

These findings help us to understand the impact of estate taxes on the long-run wealth inequality. Davies (1986) finds that the inheritance effect of the estate tax increases the long-run wealth inequality in the model with idiosyncratic labor efficiency risk.[17] This result holds for both altruistic

[17] Zhu (2016) introduces idiosyncratic investment risk into the Becker-Tomes model and finds that the inheritance effect of the estate tax reduces the long-run wealth inequality in the model with sufficiently volatile idiosyncratic investment risk.

bequest motives and "joy of giving" bequest motives, as long as the wealth accumulation equation is linear. Thus the redistribution effect plays an important role in determining the net effect of the estate tax on wealth inequality. If the redistribution effect increases wealth inequality, as in Becker and Tomes (1979) and Davies (1986), then the net effect of the estate tax increases the long-run wealth inequality. If the redistribution effect decreases wealth inequality and it dominates the inheritance effect, as in our model and Bossmann et al. (2007), then the net effect of the estate tax reduces the long-run wealth inequality.

# 4 Housing

We introduce housing into our benchmark model. We investigate the stationary distribution of individual wealth accumulation process in the steady-state aggregate economy. We will concentrate on the steady-state aggregate economy in which the aggregate capital $K$, the wage rate $w$, the interest rate $r$, the housing price $p$, and the lump-sum transfer $g$ are constant. There is 1 unit of housing in the economy. We assume that housing does not depreciate.

An agent born in period $t$ buys housing $h_{t+1}^{o}$ at the end of period $t$ and uses the housing in period $t+1$.[18] Then the agent sells the housing at the end of period $t+1$. The young agent's problem is

$$\max_{c_t^y, s_t, h_{t+1}^o, c_{t+1}^o, b_{t+1}} \log c_t^y + \beta \left( \log c_{t+1}^o + \phi \log h_{t+1}^o + \chi \log \left[ \left( 1 - \zeta \right) b_{t+1} \right] \right)$$

[18]We assume that young agents live together with their parents.

$$\begin{aligned} s.t. \quad c_t^y + s_t + p h_{t+1}^o &= w l_t + (1-\zeta) b_t + g, \\ c_{t+1}^o + b_{t+1} &= (1+r) s_t + p h_{t+1}^o, \end{aligned}$$

The agent's optimal policy functions are

$$c_{t+1}^o = \frac{1}{1+\chi} \left[ (1+r) s_t + p h_{t+1}^o \right],$$

$$b_{t+1} = \frac{\chi}{1+\chi} \left[ (1+r) s_t + p h_{t+1}^o \right],$$

$$c_t^y = \frac{1}{1+\beta(1+\chi+\phi)} \left[ w l_t + (1-\zeta) b_t + g \right],$$

$$s_t = \frac{\beta \left[ (1+\chi) r - \phi \right]}{\left[ 1+\beta(1+\chi+\phi) \right] r} \left[ w l_t + (1-\zeta) b_t + g \right],$$

and

$$h_{t+1}^o = \frac{1}{p} \frac{\beta \phi (1+r)}{\left[ 1+\beta(1+\chi+\phi) \right] r} \left[ w l_t + (1-\zeta) b_t + g \right].$$

## 4.1 The capital market

From the government's budget constraint we have $g = \zeta \int b_t di$, where $\int di$ denotes the aggregation of young agents. The capital-market clearing condition gives us

$$K_{t+1} = \int s_t di,$$

where $\int di$ denotes the aggregation of young agents. From the agent's policy functions we know that the aggregate capital follows

$$K_{t+1} = \frac{\beta \left[ (1+\chi) r - \phi \right]}{\left[ 1+\beta(1+\chi+\phi) \right] r} \left[ w + \frac{\chi r}{(1+\chi) r - \phi} (1+r) K_t \right]. \tag{17}$$

In the steady-state aggregate economy we have $K_{t+1} = K_t = K$. Thus

$w = (1-\alpha)AK^{\alpha}$ and $r = \alpha AK^{\alpha-1} - \delta$. From equation (17) we have

$$\frac{1-\alpha}{\alpha}(r+\delta)\left[(1+\chi)\,r-\phi\right]+\chi r^2-\left(1+\frac{1}{\beta}+\phi\right)r=0, \tag{18}$$

which determines the equilibrium interest rate $r$. We thus know that $r > \frac{\phi}{1+\chi}$ from equation (18).[19] Equation (18) does not depend on the estate tax $\zeta$. Thus the equilibrium interest rate $r$ does not depend on the estate tax $\zeta$. Using $r = \alpha AK^{\alpha-1} - \delta$, we have $K = \left(\frac{\alpha A}{r+\delta}\right)^{\frac{1}{1-\alpha}}$.

## 4.2 The housing market

The housing-market clearing condition is

$$\int h^o_{t+1} di = 1, \tag{19}$$

where $\int di$ denotes the aggregation of old agents.

From the agent's policy functions we have

$$ph^o_{t+1} = \frac{(1+r)\phi}{(1+\chi)\,r-\phi}s_t.$$

Using the housing-market clearing condition (19) we have

$$p = \frac{(1+r)\phi}{(1+\chi)\,r-\phi}K.$$

[19]The negative root of equation (18) cannot be the equilibrium interest rate in the economy with housing. The agent has two ways of holding assets from the young period to the old period, savings and housing. We assume that housing does not depreciate. And the housing price does not change in the stationary equilibrium. Thus the return of housing is positive. There exist aribitrages if the interest rate of saving is negative.

## 4.3 The wealth distribution

Let $a_{t+1} = s_t + ph^o_{t+1}$. Now one component of the wealth is saving. We include housing as the other component of the wealth. From the agent's policy functions we have the individual wealth accumulation equation,

$$a_{t+1} = c_9 l_t + c_{10}\left[(1-\zeta)a_t + \zeta\bar{W}\right], \tag{20}$$

where $c_9 = \frac{\beta(1+\chi+\phi)}{1+\beta(1+\chi+\phi)}w$ and $c_{10} = \frac{\beta\chi}{1+\beta(1+\chi+\phi)}(1+r)$. $\bar{W}$ denotes the aggregate wealth of the economy,

$$\bar{W} = \int a_t di = K + p.$$

Note that $c_9$, $c_{10}$, and $\bar{W}$ do not depend on the estate tax $\zeta$.

From equation (20) we have the long-run wealth distribution,

$$a_\infty =_{st} \sum_{s=0}^{\infty} c_{11}^s \left(c_9 l_s + c_{10}\zeta\bar{W}\right),$$

where $c_{11} = c_{10}(1-\zeta)$.

Comparing equations (20) and (15) we find that all the theoretical results of the long-run wealth inequality in the benchmark model still hold in a model with housing. We can decompose the impact of the estate tax on the long-run wealth inequality into two channels, the inheritance effect and the redistribution effect. The channel through which the estate tax influences $c_{11}$ is the inheritance effect. The higher $\zeta$ the lower $c_{11}$. Thus the inheritance effect increases the long-run wealth inequality. The channel through which the estate tax influences the term $c_9 l_s + c_{10}\zeta\bar{W}$ is the redistribution effect. The higher $\zeta$ the higher $\zeta\bar{W}$. Thus the redistribution effect reduces the long-run wealth inequality. As in Theorem 2, the net effect is that the estate tax $\zeta$ reduces the long-run wealth inequality.

# 5 A life-cycle model

We investigate the stationary distribution of individual wealth accumulation process in the steady-state aggregate economy. We will concentrate on the steady-state aggregate economy in which the aggregate capital $K$, the wage rate $w$, the interest rate $r$, and the lump-sum transfer $g$ are constant.

We normalize the population of the economy to 1. Agents live for $T+1$ periods. At the end of age $T+1$, the agent dies and gives birth to one child. The retirement age is $R$. At the beginning of the life, the agent of dynasty $n$ draws his labor efficiency $l_n$. Then the agent keeps this labor efficiency for the whole life. We assume that $\{l_n\}$ is *i.i.d.* along generations. The agent's problem is

$$\max_{c_t^y, s_t, h_{t+1}^o, c_{t+1}^o, b_{t+1}} \sum_{\tau=0}^{T} \beta^{\tau} \frac{(c_{n,\tau})^{1-\eta} - 1}{1-\eta} + \beta^T \chi \frac{[(1-\zeta)\, b_{n+1}]^{1-\eta} - 1}{1-\eta}$$

$$\begin{aligned}
s.t. \quad a_{n,1} + c_{n,0} &= w l_n + (1-\zeta)\, b_n + g, \\
a_{n,\tau+1} + c_{n,\tau} &= (1+r) a_{n,\tau} + w l_n, \ 1 \le \tau \le R-1, \\
a_{n,\tau+1} + c_{n,\tau} &= (1+r) a_{n,\tau}, \ R \le \tau \le T-1, \\
c_{n,T} + b_{n+1} &= (1+r) a_{n,T}.
\end{aligned}$$

The agent's optimal policy functions during the retirement periods are

$$b_{n+1} = \frac{\chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}}}{1+\chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}}} (1+r) a_{n,T},$$

$$c_{n,T} = \frac{1}{1+\chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}}} (1+r) a_{n,T}.$$

$$a_{n,\tau+1} = \frac{\tilde{\beta}_{\tau+1}^{\frac{1}{\eta}}}{1+\tilde{\beta}_{\tau+1}^{\frac{1}{\eta}}} (1+r) a_{n,\tau}, \; R \leq \tau \leq T-1,$$

and

$$c_{n,\tau} = \frac{1}{1+\tilde{\beta}_{\tau+1}^{\frac{1}{\eta}}} (1+r) a_{n,\tau}, \; R \leq \tau \leq T-1,$$

where $\tilde{\beta}_T = \beta \left[1+\chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}}\right]^{\eta} (1+r)^{1-\eta}$ and $\tilde{\beta}_\tau = \beta \left(1+\tilde{\beta}_{\tau+1}^{\frac{1}{\eta}}\right)^{\eta} (1+r)^{1-\eta}$ for $R \leq \tau \leq T-1$.

The agent's optimal policy functions during the work periods are

$$c_{n,0} = \frac{1}{1+\tilde{\beta}_1^{\frac{1}{\eta}}} \left[(1-\zeta) b_n + g + \left(1+\frac{1}{r}\right)\left(1-\frac{1}{(1+r)^R}\right) w l_n\right],$$

$$a_{n,1} = (1-\zeta) b_n + g + w l_n - c_{n,0},$$

$$c_{n,\tau} = \frac{1+r}{1+\tilde{\beta}_{\tau+1}^{\frac{1}{\eta}}} \left[a_{n,\tau} + \frac{1}{r}\left(1-\frac{1}{(1+r)^{R-\tau}}\right) w l_n\right], \; 1 \leq \tau \leq R-1,$$

and

$$a_{n,\tau+1} = (1+r) a_{n,\tau} + w l_n - c_{n,\tau}, \; 1 \leq \tau \leq R-1,$$

where $\tilde{\beta}_\tau = \beta \left(1+\tilde{\beta}_{\tau+1}^{\frac{1}{\eta}}\right)^{\eta} (1+r)^{1-\eta}$ for $1 \leq \tau \leq R-1$.

From the government's budget constraint we have

$$g = \zeta \int b_n di,$$

where $\int di$ denotes the aggregation of age 0 agents.

The capital-market clearing condition gives us

$$K = \frac{1}{T}\sum_{\tau=1}^{T}\int a_\tau di,$$

where $\int a_\tau di$ denotes the aggregate capital within the age $\tau$ cohort.

The aggregate population of workers is 1. And we assume that $E(l_n) = 1$. Thus the labor-market clearing condition is

$$L = \frac{R}{T}.$$

In the steady-state aggregate economy we have

$$w = (1-\alpha)AK^{\alpha}L^{-\alpha},$$

and

$$r = \alpha AK^{\alpha-1}L^{1-\alpha} - \delta.$$

To illustrate the impact of the estate tax $\zeta$ on the long-run wealth inequality, we implement a simple calibration exercise. We pick $T = 60$, $R = 40$, $\beta = 0.95$, $\eta = 2$, $\chi = 0.8$, $A = 1$, $\alpha = \frac{1}{3}$, and $\delta = 0.05$. We assume that $l_t \sim U[0, 2]$.[20] Thus $E(l_t) = 1$. Table 4 reports the simulation results of the life-cycle model.

Table 4: The impact of the estate tax on the long-run wealth inequality

[20] The Gini coefficient of the earnings distribution is 0.33. The Gini coefficients of the long-run wealth distribution in Table 4 are larger than this number since there exists a life-cyle pattern of savings.

| $\zeta$ | $K$ | $g$ | $\int b_n di$ | $Gini$ |
|---|---|---|---|---|
| 0 | 5.440 | 0 | 0.691 | 0.525 |
| 0.1 | 5.452 | 0.073 | 0.727 | 0.523 |
| 0.2 | 5.476 | 0.155 | 0.769 | 0.520 |
| 0.3 | 5.506 | 0.247 | 0.819 | 0.518 |
| 0.4 | 5.541 | 0.352 | 0.881 | 0.514 |
| 0.5 | 5.596 | 0.479 | 0.957 | 0.510 |
| 0.6 | 5.651 | 0.638 | 1.064 | 0.505 |

Table 4 shows that the higher the estate tax $\zeta$ the lower the Gini coefficient of the long-run wealth distribution. The estate tax reduces the long-run wealth inequality. Table 4 also shows that the higher the estate tax $\zeta$ the higher $g$. Thus the redistribution effect of the estate tax decreases the long-run wealth inequality. The results of the long-run wealth inequality in our benchmark model are still true in this life-cycle model.

# 6 Conclusion

Bossmann et al. (2007) find that estate taxes reduce the long-run wealth inequality. This result contrasts with the findings of previous literatures with idiosyncratic labor efficiency risk, such as Becker and Tomes (1979) and Davies (1986). These papers show that estate taxes usually increase the long-run wealth inequality. In this paper we use the decomposition technique developed by Davies (1986) to reinvestigate the impact of estate taxes on the long-run wealth inequality. We find that the redistribution effect plays an important role in determining the effect of the estate tax on the long-run wealth inequality. We also extend our benchmark model in two directions. In the first extension, we include housing as a new asset in the model. In the other extension, we permit the agent to live for more than two periods. In these extensions we show that the results of the long-run wealth inequality in our benchmark model are still true.

Our findings also help us to understand how different ways of modelling bequest motives influence the impact of estate taxes on wealth inequality. Different forms of bequest motives, altruism and "joy of giving", do not influence the inheritance effect of the estate tax, but they imply different redistribution effects of the estate tax. Thus different forms of bequest motives influence the impact of estate taxes on wealth inequality through the redistribution effect.

# References


[1] Aiyagari, S. R. (1994): "Uninsured idiosyncratic risk and aggregate saving," *Quarterly Journal of Economics*, 109, 659-684.

[2] Arnold, B. (1987): *Majorization and the Lorenz order: A brief introduction*, Berlin: Springer-Verlag.

[3] Atkinson, A. (1970): "On the measurement of inequality," *Journal of Economic Theory*, 2, 244-263.

[4] Becker, G., and N. Tomes (1979): "An equilibrium theory of the distribution of income and intergenerational mobility," *Journal of Political Economy*, 87, 1153-1189.

[5] Benhabib, J., A. Bisin, and S. Zhu (2011): "The distribution of wealth and fiscal policy in economies with finitely lived agents," *Econometrica*, 79, 123-157.

[6] Benhabib, J., A. Bisin, and S. Zhu (2015): "The wealth distribution in Bewley economies with capital income risk," *Journal of Economic Theory*, 159, 489-515.

[7] Bossmann, M., C. Kleiber, and K. Walde (2007): "Bequest, taxation and the distribution of wealth in a general equilibrium model," *Journal of Public Economics*, 91, 1247-1271.

[8] Brandt, A. (1986): "The stochastic equation $Y_{n+1} = A_n Y_n + B_n$ with stationary coefficients," *Advances in Applied Probability*, 18, 211-220.

[9] Castaneda, A., J. Diaz-Gimenez, and J-V. Rios-Rull (2003): "Accounting for the U.S. earnings and wealth inequality," *Journal of Political Economy*, 111, 818-857.

[10] Chatterjee, S. (1994): "Transitional dynamics and the distribution of wealth in a neoclassical growth model," *Journal of Public Economics*, 54, 97-119.

[11] Davies, J. (1986): "Does redistribution reduce inequality?" *Journal of Labor Economics*, 4, 538-559.

[12] Davies, J., and P. Kuhn (1991): "A dynamic model of redistribution, inheritance, and inequality," *Canadian Journal of Economics*, 24, 324-344.

[13] De Nardi, M. (2004): "Wealth inequality and intergenerational links," *Review of Economic Studies*, 71, 743-768.

[14] De Nardi, M., and F. Yang (2016): "Wealth inequality, family background, and estate taxation," *Journal of Monetary Economics*, 77, 130-145.

[15] Fellman, J. (1976): "The effect of transformations on Lorenz curves," *Econometrica*, 44, 823-824.

[16] Gajdos, T., and J. Weymark (2012): "Introduction to inequality and risk," *Journal of Economic Theory*, 147, 1313-1330.

[17] Gale, W., and M. Perozek (2001): "Do estate taxes reduce saving?" in *Rethinking Estate and Gift Taxation*, ed. by W. Gale, J. Hines Jr., and J. Slemrod. Washington D.C.: Brookings Institution Press.

[18] Gastwirth, J. (1971): "A general definition of the Lorenz curve," *Econometrica*, 39, 1037-1039.

[19] Kopczuk, W. (2013): "Taxation of intergenerational transfers and wealth," in *Handbook of Public Economics*, ed. by A. Auerbach, R. Chetty, M. Feldstein, and E. Saez. Amsterdam: Elsevier.

[20] Marshall, A., and I. Olkin (2007): *Life Distributions*, New York, NY: Springer.

[21] Rothschild, M., and J. Stiglitz (1973): "Some further results on the measurement of inequality," *Journal of Economic Theory*, 6, 188-204.

[22] Shaked, M., and G. Shanthikumar (2010): *Stochastic Orders*, New York, NY: Springer.

[23] Solon, G. (1992): "Intergenerational income mobility in the United States," *American Economic Review*, 82, 393-408.

[24] Zhu, S. (2016): "A Becker-Tomes model with investment risk," Mimeo, National University of Singapore.

[25] Zilcha, I. (2003): "Intergenerational transfers, production and income distribution," *Journal of Public Economics*, 87, 489-513.

[26] Zimmerman, D. (1992): "Regression toward mediocrity in economic stature," *American Economic Review*, 82, 409-429.

# A Appendix

## A.1 Proof of Proposition 1

Proof: Letting $K_{t+1} = K_t = K$ in equation (11) we have

$$K = \left( \frac{1 - \alpha + \varphi\alpha}{1 + \tilde{\beta}^{-\frac{1}{\eta}} - \varphi(1-\delta)} A \right)^{\frac{1}{1-\alpha}}, \tag{A.1}$$

where $\tilde{\beta} = \beta \left[ 1 + \chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}} \right]^{\eta} (1+r)^{1-\eta}$ and $r = \alpha A K^{\alpha-1} - \delta$.

Plugging equation (A.1) into $r = \alpha A K^{\alpha-1} - \delta$ we have

$$\frac{r + \delta}{\alpha} = \frac{1 + \tilde{\beta}^{-\frac{1}{\eta}} - \varphi(1-\delta)}{1 - \alpha + \varphi\alpha}. \tag{A.2}$$

Plugging $\tilde{\beta} = \beta \left[ 1 + \chi^{\frac{1}{\eta}} (1-\zeta)^{\frac{1-\eta}{\eta}} \right]^{\eta} (1+r)^{1-\eta} = \frac{\beta}{(1-\varphi)^{\eta}} (1+r)^{1-\eta}$ into equation (A.2) we have

$$\frac{1-\alpha}{\alpha}(r+\delta) + \varphi(1+r) - (1-\varphi)\beta^{-\frac{1}{\eta}}(1+r)^{1-\frac{1}{\eta}} = 1.$$

We show Proposition 1 in two cases:

Case (i) $\eta > 1$

Note that $0 < \varphi < 1$. Define

$$h(\varphi, r) = \frac{1-\alpha}{\alpha}(r+\delta) + \varphi(1+r) - (1-\varphi)\beta^{-\frac{1}{\eta}}(1+r)^{1-\frac{1}{\eta}}.$$

The equilibrium $r$ is determined by

$$h(\varphi, r) = 1.$$

Note that $h(\varphi, r)$ is a continuous function of $r$, with

$$h(\varphi, -\delta) = \varphi(1-\delta) - (1-\varphi)\beta^{-\frac{1}{\eta}}(1-\delta)^{1-\frac{1}{\eta}} < \varphi(1-\delta) < 1$$

and

$$\lim_{r\to\infty} h(\varphi, r) = \infty$$

Also $h_{22}(\varphi, r) = \left(1 - \frac{1}{\eta}\right)\frac{1}{\eta}(1-\varphi)\beta^{-\frac{1}{\eta}}(1+r)^{-\frac{1}{\eta}-1} > 0$ due to $\eta > 1$. Thus $h(\varphi, r)$ is a strictly convex function of $r$. Therefore there must exist a unique equilibrium $r > -\delta$.[21]

Note that $h(\varphi, r)$ is strictly increasing in $\varphi$. For $\varphi_1 < \varphi_2 < 1$, suppose that

$$h(\varphi_1, r_1) = 1 \quad \text{and} \quad h(\varphi_2, r_2) = 1.$$

We have

$$h(\varphi_2, r_1) > h(\varphi_1, r_1) = 1.$$

Thus $r_2 < r_1$ since $h(\varphi_2, -\delta) < 1$ and $h(\varphi_2, r)$ is a continuous function of $r$. A higher $\chi$ implies a higher $\varphi$. Thus a higher $\chi$ implies a lower $r$ and a higher $K$.

Case (ii) $\eta = 1$

In this case $\tilde{\beta} = \beta(1+\chi)$ and $\varphi = \frac{1}{1+\frac{1}{\chi}}$, equation (A.1) implies

$$\begin{aligned} K &= \left(\frac{1-\alpha+\chi}{1+\frac{1}{\beta}+\delta\chi}A\right)^{\frac{1}{1-\alpha}} \\ &= \left(\left[\frac{1}{\delta} - \frac{\frac{1}{\delta}\left(1+\frac{1}{\beta}\right)-(1-\alpha)}{1+\frac{1}{\beta}+\delta\chi}\right]A\right)^{\frac{1}{1-\alpha}}. \end{aligned}$$

[21] In the equilibrium $r$ could be negative. Since saving is the only way to bring wealth to the next period, even if $r$ is negative, the agent still saves.

Thus a higher $\chi$ implies a higher $K$. ■

## A.2 Proof of Proposition 2

Proof: Obviously $c_4 \geq 0$. From equation (A.2) we have

$$1+r=\frac{\left(1+\tilde{\beta}^{-\frac{1}{\eta}}\right)\alpha+(1-\delta)(1-\alpha)}{(1-\alpha)+\varphi\alpha}$$

Thus

$$c_4=\frac{(1-\zeta)\,\varphi(1+r)}{1+\tilde{\beta}^{-\frac{1}{\eta}}}=(1-\zeta)\,\frac{\alpha+\frac{1-\delta}{1+\tilde{\beta}^{-\frac{1}{\eta}}}(1-\alpha)}{\alpha+\frac{1}{\varphi}(1-\alpha)}<1$$

since $\tilde{\beta}=\beta\left[1+\chi^{\frac{1}{\eta}}(1-\zeta)^{\frac{1-\eta}{\eta}}\right]^{\eta}(1+r)^{1-\eta}>0$ and $0<\varphi<1$. ■

## A.3 Proof of Proposition 3

Proof: From equation (13) we have

$$a_{t+1}=c_3 l_t+c_4 a_t+c_5,$$

where $c_3=\frac{1}{1+\tilde{\beta}^{-\frac{1}{\eta}}}w$, $c_4=\frac{(1-\zeta)\varphi(1+r)}{1+\tilde{\beta}^{-\frac{1}{\eta}}}$, and $c_5=\frac{\zeta\varphi(1+r)}{1+\tilde{\beta}^{-\frac{1}{\eta}}}K$. Let $B_t=c_5+c_3 l_t$. We have

$$a_{t+1}=c_4 a_t+B_t. \tag{A.3}$$

Note that $\{B_t\}$ is stationary and ergodic since $\{l_t\}$ is stationary and ergodic by Assumption 1. We have $-\infty\leq\log c_4<0$. Also $E(B_t)=c_5+c_3<\infty$, since $E(l_t)=1$ by Assumption 2. Thus $E(\log B_t)^+\leq E(B_t)<\infty$. By Theorem 1 of Brandt (1986) we know that $a_t$ converges to $\sum_{j=0}^{\infty}c_4^j B_{t-j-1}$ almost surely as $t$ approaches infinity. Thus $a_t\to_{st}\sum_{j=0}^{\infty}c_4^j B_{t-j-1}$ as $t$ approaches infinity.

Since $\{B_t\}$ is stationary, we know that the sequence of $(B_{t-1}, B_{t-2}, \cdots, B_{t-j-1}, \cdots)$ has the same distribution as the sequence of $(B_0, B_1, \cdots, B_s, \cdots)$. Thus we have

$$\sum_{j=0}^{\infty} c_4^j B_{t-j-1} =_{st} \sum_{s=0}^{\infty} c_4^s B_s, \ \forall t \in \mathbb{Z}.$$

Let

$$a_\infty =_{st} \sum_{s=0}^{\infty} c_4^s B_s =_{st} c_3 \sum_{s=0}^{\infty} c_4^s l_s + \frac{c_5}{1 - c_4}.$$

Thus we know that $a_t \rightarrow_{st} a_\infty$ as $t$ approaches infinity. ■

## A.4 Proof of Theorem 1

Proof:

Theorem 3.A.36 of Shaked and Shanthikumar (2010) shows that

**Lemma 2** *Let* $X_1, X_2, \cdots, X_n$ *and* $Y$ *be* $n+1$ *random variables. If* $X_i \preceq_{cx} Y$, $i = 1, 2, \cdots, n$, *then*

$$\sum_{i=1}^{n} a_i X_i \preceq_{cx} Y,$$

*whenever* $a_i \geq 0$, $i = 1, 2, \cdots, n$, *and* $\sum_{i=1}^{n} a_i = 1$.[22]

Theorem 3.A.10 of Shaked and Shanthikumar (2010) states that

**Lemma 3** *Let* $X$ *and* $Y$ *be two non-negative random variables with equal means. Then* $X \preceq_{cx} Y$ *if and only if* $L_X(p) \geq L_Y(p)$ *for all* $p \in [0, 1]$.

[22] For two random variables $X$ and $Y$, $X$ is smaller than $Y$ in the convex order, denoted by $X \preceq_{cx} Y$, if and only if

$$E[\phi(X)] \leq E[\phi(Y)]$$

for all convex functions $\phi : \mathbb{R} \rightarrow \mathbb{R}$, provided the expectations exist. For more properties of the convex order, see Shaked and Shanthikumar (2010).

Note that $a_\infty^A$ has the same Lorenz curve as $l_1$. We only need to show that $a_\infty^B \succeq_L l_1$.

In economy $B$, pick $a_1 = \frac{c_3}{1-c_4}$.[23] Thus

$$a_1 \preceq_{cx} \frac{c_3}{1-c_4} l_1$$

since $a_1 = E\left(\frac{c_3}{1-c_4} l_1\right)$.[24]

Suppose that

$$a_t \preceq_{cx} \frac{c_3}{1-c_4} l_1.$$

Thus $\frac{1-c_4}{c_3} a_t \preceq_{cx} l_1$.[25]

And

$$\begin{aligned} a_{t+1} &= c_3 l_t + c_4 a_t \\ &= \frac{c_3}{1-c_4}\left((1-c_4) l_t + c_4 \frac{1-c_4}{c_3} a_t\right). \end{aligned}$$

Note that $(1-c_4) l_t + c_4 \frac{1-c_4}{c_3} a_t$ is a weighted average of $l_t$ and $\frac{1-c_4}{c_3} a_t$. For $\forall t \geq 1$, $l_t$ and $l_1$ have the same distribution. We have $l_t \preceq_{cx} l_1$, $\forall t \geq 1$. By Lemma 2 we have

$$(1-c_4) l_t + c_4 \frac{1-c_4}{c_3} a_t \preceq_{cx} l_1.$$

Thus

$$a_{t+1} \preceq_{cx} \frac{c_3}{1-c_4} l_1.$$

---

[23] We abuse notations a little bit. We use $a_t$ instead of $a_t^B$ without confusions.

[24] Let $X$ be a random variable with a finte mean. $E(X) \preceq_{cx} X$ can be established by applying Jensen's Inequality and the definition of the convex order.

[25] $X \preceq_{cx} Y$ implies $bX \preceq_{cx} bY$ for any $b \in \mathbb{R}$. Note that $\phi(bx)$ is a convex function of $x \in \mathbb{R}$ if $\phi(x)$ is a convex function of $x \in \mathbb{R}$.

By mathematical induction we have

$$a_t \preceq_{cx} \frac{c_3}{1-c_4} l_1, \quad \forall t \geq 1.$$

Since $a_t \rightarrow_{st} a_\infty^B$ as $t$ approaches infinity, we have

$$a_\infty^B \preceq_{cx} \frac{c_3}{1-c_4} l_1,$$

by part (c) of Theorem 3.A.12 of Shaked and Shanthikumar (2010). By Lemma 3 we have $a_\infty^B \succeq_L \frac{c_3}{1-c_4} l_1$ since $E\left(a_\infty^B\right) = E\left(\frac{c_3}{1-c_4} l_1\right) = \frac{c_3}{1-c_4}$. Thus $a_\infty^B \succeq_L l_1$. ■

## A.5 Proof of Lemma 1

Proof: Let

$$g(x) = (1-\zeta)x + \zeta E\left(X\right), \quad x \in [0, +\infty)$$

and

$$h(x) = (1-\hat{\zeta})x + \hat{\zeta} E\left(X\right), \quad x \in [0, +\infty)$$

Note that $g(\cdot)$ and $h(\cdot)$ are non-negative increasing functions defined on $[0, +\infty)$, since $0 \leq \hat{\zeta} \leq \zeta < 1$. Also $g(x) > 0$ and $h(x) > 0$ for $x > 0$. Note that $\frac{h(x)}{g(x)}$ is increasing in $x \in (0, +\infty)$, since

$$\begin{aligned} \frac{h(x)}{g(x)} &= \frac{(1-\hat{\zeta})x + \hat{\zeta} E\left(X\right)}{(1-\zeta)x + \zeta E\left(X\right)} \\ &= \frac{1-\hat{\zeta}}{1-\zeta}\left[1 - \frac{\zeta - \hat{\zeta}}{\left(1-\hat{\zeta}\right)(1-\zeta)} \frac{E(X)}{x + \frac{\zeta}{1-\zeta} E(X)}\right]. \end{aligned}$$

By Theorem 3.A.26 of Shaked and Shanthikumar (2010) we have $g(X) \succeq_L h(X)$, i.e. $(1-\zeta)X+\zeta E(X) \succeq_L (1-\hat{\zeta})X+\hat{\zeta}E(X)$. By Lemma 3 we have $(1-\zeta)X+\zeta E(X) \preceq_{cx} (1-\hat{\zeta})X+\hat{\zeta}E(X)$ since $E[(1-\zeta)X+\zeta E(X)] = E(X) = E\left[(1-\hat{\zeta})X+\hat{\zeta}E(X)\right]$. ■

## A.6 Proof of Theorem 2

Proof: Note that $a_\infty^\zeta$ is the stationary distribution of the stochastic process $\{a_t^\zeta\}$ which is generated by

$$a_{t+1}^\zeta = c_6 l_t + c_7\left[(1-\zeta)a_t^\zeta + \zeta\bar{K}\right]$$

and a given $a_1^\zeta$. And $a_\infty^{\hat{\zeta}}$ is the stationary distribution of the stochastic process $\{a_t^{\hat{\zeta}}\}$ which is generated by

$$a_{t+1}^{\hat{\zeta}} = c_6 l_t + c_7\left[(1-\hat{\zeta})a_t^{\hat{\zeta}} + \hat{\zeta}\bar{K}\right]$$

and a given $a_1^{\hat{\zeta}}$.

Let $a_1^\zeta =_{st} a_1^{\hat{\zeta}}$. Thus $a_1^\zeta \preceq_{cx} a_1^{\hat{\zeta}}$ by the definition of the convex order.

Now suppose that $a_t^\zeta \preceq_{cx} a_t^{\hat{\zeta}}$. By Lemma 1 we have

$$(1-\zeta)a_t^\zeta + \zeta\bar{K} \preceq_{cx} (1-\hat{\zeta})a_t^\zeta + \hat{\zeta}\bar{K}$$

since $E\left(a_t^\zeta\right) = \bar{K}$.

By Corollary 3.A.22 of Shaked and Shanthikumar (2010) we have $(1-\hat{\zeta})a_t^\zeta \preceq_{cx} (1-\hat{\zeta})a_t^{\hat{\zeta}}$ since $\left(1-\hat{\zeta}\right)$ is independent of $a_t^\zeta$ and $a_t^{\hat{\zeta}}$. By Part (d) of Theorem 3.A.12 of Shaked and Shanthikumar (2010) we have

$$(1-\hat{\zeta})a_t^{\zeta}+\hat{\zeta}\bar{K} \preceq_{cx} (1-\hat{\zeta})a_t^{\hat{\zeta}}+\hat{\zeta}\bar{K},$$

since $\hat{\zeta}\bar{K}$ is independent of $(1-\hat{\zeta})a_t^{\zeta}$ and $(1-\hat{\zeta})a_t^{\hat{\zeta}}$. By the transitivity of the convex order we have

$$(1-\zeta)a_t^{\zeta}+\zeta\bar{K} \preceq_{cx} (1-\hat{\zeta})a_t^{\hat{\zeta}}+\hat{\zeta}\bar{K}.$$

Thus we have $c_7\left[(1-\zeta)a_t^{\zeta}+\zeta\bar{K}\right] \preceq_{cx} c_7\left[(1-\hat{\zeta})a_t^{\hat{\zeta}}+\hat{\zeta}\bar{K}\right]$ by the property of the convex order in Footnote 21. Note that $c_6 l_t$ and $c_7\left[(1-\zeta)a_t^{\zeta}+\zeta\bar{K}\right]$ are independent. And $c_6 l_t$ and $c_7\left[(1-\hat{\zeta})a_t^{\hat{\zeta}}+\hat{\zeta}\bar{K}\right]$ are independent. Thus by part (d) of Theorem 3.A.12 of Shaked and Shanthikumar (2010), we have

$$c_6 l_t + c_7\left[(1-\zeta)a_t^{\zeta}+\zeta\bar{K}\right] \preceq_{cx} c_6 l_t + c_7\left[(1-\hat{\zeta})a_t^{\hat{\zeta}}+\hat{\zeta}\bar{K}\right].$$

Thus we have

$$a_{t+1}^{\zeta} \preceq_{cx} a_{t+1}^{\hat{\zeta}}.$$

By mathematical induction we have

$$a_t^{\zeta} \preceq_{cx} a_t^{\hat{\zeta}},\ \forall t \geq 1.$$

Since $a_t^{\zeta} \rightarrow_{st} a_{\infty}^{\zeta}$ and $a_t^{\hat{\zeta}} \rightarrow_{st} a_{\infty}^{\hat{\zeta}}$ as $t$ approaches infinity, we have

$$a_{\infty}^{\zeta} \preceq_{cx} a_{\infty}^{\hat{\zeta}},$$

by part (c) of Theorem 3.A.12 of Shaked and Shanthikumar (2010). By lemma 3 we have

$$a_\infty^\zeta \succeq_L a_\infty^{\hat{\zeta}},$$

since $E\left(a_\infty^\zeta\right) = E\left(a_\infty^{\hat{\zeta}}\right) = \bar{K}$. ■

## A.7 An alternative set-up of the model

Here we investigate an alternative set-up of our benchmark model. The main difference is that $b_t$ In our benchmark model is the before-tax bequest. In this alternative set-up, $b_t$ is the after-tax bequest.

The agent's problem is

$$\max_{c_t^y, s_t, c_{t+1}^o, b_{t+1}} \log c_t^y + \beta\left(\log c_{t+1}^o + \chi \log b_{t+1}\right)$$

$$\begin{aligned} s.t. \quad c_t^y + s_t &= w_t l_t + b_t + g_t, \\ c_{t+1}^o + (1+\zeta)\, b_{t+1} &= (1+r_{t+1}) s_t. \end{aligned}$$

The agent's optimal policy functions are

$$c_{t+1}^o = \frac{1}{1+\chi}(1+r_{t+1})s_t,$$

$$b_{t+1} = \frac{\chi}{(1+\chi)(1+\zeta)}(1+r_{t+1})s_t,$$

$$c_t^y = \frac{1}{1+\beta(1+\chi)}\left(w_t l_t + b_t + g_t\right),$$

and

$$s_t = \frac{\beta(1+\chi)}{1+\beta(1+\chi)}\left(w_t l_t + b_t + g_t\right).$$

From the government's budget constraint we have

$$g_t = \zeta \int b_t di,$$

where $\int di$ denotes the aggregation of old agents.

Thus the aggregate capital follows

$$\begin{aligned} K_{t+1} &= \int s_t di \\ &= \frac{\beta(1+\chi)}{1+\beta(1+\chi)}\left[w_t + \frac{\chi}{1+\chi}(1+r_t)K_t\right], \end{aligned}$$

where $w_t = (1-\alpha)AK_t^{\alpha}$ and $r_t = \alpha AK_t^{\alpha-1} - \delta$.

In the steady-state aggregate economy we have $K_{t+1} = K_t = \bar{K}$. Thus we have

$$\bar{K} = \left(\frac{1-\alpha+\chi}{1+\frac{1}{\beta}+\delta\chi}A\right)^{\frac{1}{1-\alpha}}.$$

The estate tax does not affect the aggregate capital. Then $\bar{w} = (1-\alpha)A\left(\bar{K}\right)^{\alpha}$ and $\bar{r} = \alpha A\left(\bar{K}\right)^{\alpha-1} - \delta$.

Let $a_{t+1} = s_t$. From the agent's policy functions we have the individual wealth accumulation equation,

$$a_{t+1} = c_6 l_t + c_7\left[\frac{1}{1+\zeta}a_t + \frac{\zeta}{1+\zeta}\bar{K}\right], \tag{21}$$

with $c_6 = \frac{1}{1+\frac{1}{\beta(1+\chi)}}\bar{w}$ and $c_7 = \frac{1}{\left(1+\frac{1}{\beta(1+\chi)}\right)\left(1+\frac{1}{\chi}\right)}(1+\bar{r})$. Both $c_6$ and $c_7$ do not depend on the estate tax $\zeta$.

From equation (21) we have the long-run wealth distribution,

$$a_{\infty} =_{st} \sum_{s=0}^{\infty}(\tilde{c}_8)^s\left(c_6 l_s + c_7\frac{\zeta}{1+\zeta}\bar{K}\right),$$

where $\tilde{c}_8 = c_7 \frac{1}{1+\zeta}$. Comparing equations (21) and (15) we find that all the theoretical results of the long-run wealth inequality in the benchmark model still hold in this alternative set-up.

## A.8 The Becker-Tomes model

Here we briefly review some main results of Becker-Tomes models by Becker and Tomes (1979) and Davies (1986). As in Becker and Tomes (1979) and Davies (1986), we assume that each agent only lives for one period. At the end of the period, the agent dies and gives birth to one child. The prices of $r$ and $w$ are exogenous and constant. Davies (1986) explains that the aim of using exogenous prices of $r$ and $w$ in his paper is exactly to close the general equilibrium effect of the estate tax.[26]

As in Becker and Tomes (1979) and Davies (1986) we assume that the agent can correctly anticipate the labor efficiency of his child. The agent's problem is

$$\max_{c_t, b_{t+1}, I_{t+1}} \frac{c_t^{1-\eta} - 1}{1-\eta} + \chi \frac{I_{t+1}^{1-\eta} - 1}{1-\eta}$$

$$s.t. \quad c_t + b_{t+1} = I_t,$$

$$I_{t+1} = wl_{t+1} + (1+r)(1-\zeta) b_{t+1} + g,$$

where $I_{t+1}$ is the total wealth of the child. The agent's optimal policy

[26] After we solve the general equilibrium in our benchmark model with 'joy of giving' bequest motives, the estate tax does not affect the prices of $r$ and $w$ when utility functions are logarithmic. Thus the estate tax does not have a general equilibrium effect. However, the estate tax does affect the prices of $r$ and $w$ in a model with altruistic bequest motives even for logarithmic utility functions. Then the estate tax does have a general equilibrium effect. Thus a model with altruistic bequest motives and endogenous prices of $r$ and $w$ is not comparable to our benchmark model.

We then decide to follow the literatures of Becker and Tomes (1979) and Davies (1986) to assume that the prices of $r$ and $w$ are exogenous. Thus we can concentrate on the inheritance effect and the redistribution effect of the estate tax.

functions are

$$c_t = \frac{1}{1 + \left[(1+r)(1-\zeta)\right]^{\frac{1-\eta}{\eta}} \chi^{\frac{1}{\eta}}} \left( I_t + \frac{wl_{t+1} + g}{(1+r)(1-\zeta)} \right),$$

$$b_{t+1} = \frac{1}{1 + \left[(1+r)(1-\zeta)\right]^{\frac{\eta-1}{\eta}} \chi^{-\frac{1}{\eta}}} I_t - \frac{1}{1 + \left[(1+r)(1-\zeta)\right]^{\frac{1-\eta}{\eta}} \chi^{\frac{1}{\eta}}} \frac{wl_{t+1} + g}{(1+r)(1-\zeta)},$$

and

$$I_{t+1} = \frac{(1+r)(1-\zeta)}{1 + \left[(1+r)(1-\zeta)\right]^{\frac{\eta-1}{\eta}} \chi^{-\frac{1}{\eta}}} \left( I_t + \frac{wl_{t+1} + g}{(1+r)(1-\zeta)} \right). \tag{22}$$

From the government's budget constraint we have

$$g = \zeta(1+r) \int b_t di,$$

where $\int di$ denotes the aggregation of young agents.

In the steady-state aggregate economy we have $\int I_{t+1} di = \int I_t di = \bar{I}$. Thus we have

$$\bar{I} = \int I_t di = w + (1+r)(1-\zeta) \int b_t di + g = w + \frac{g}{\zeta}. \tag{23}$$

From equation (22) we have

$$\bar{I} = \frac{(1+r)(1-\zeta)}{1 + \left[(1+r)(1-\zeta)\right]^{\frac{\eta-1}{\eta}} \chi^{-\frac{1}{\eta}}} \left( \bar{I} + \frac{w + g}{(1+r)(1-\zeta)} \right). \tag{24}$$

Combining equations (23) and (24) we have

$$\bar{I} = \frac{1}{1 - (1+r)\left(1 - \left[(1+r)(1-\zeta)\right]^{-\frac{1}{\eta}} \chi^{-\frac{1}{\eta}}\right)} w,$$

and

$$g = mw,$$

where $m = \frac{\zeta}{\frac{1}{(1+r)\left(1-[(1+r)(1-\zeta)]^{-\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}\right)}-1}$.

From equation (22) we have the individual wealth accumulation equation,

$$I_{t+1} = c_{12} I_t + \frac{1}{1+\left[(1+r)(1-\zeta)\right]^{\frac{\eta-1}{\eta}}\chi^{-\frac{1}{\eta}}}\left(wl_{t+1}+g\right), \tag{25}$$

where $c_{12} = \frac{(1+r)(1-\zeta)}{1+[(1+r)(1-\zeta)]^{\frac{\eta-1}{\eta}}\chi^{-\frac{1}{\eta}}}$.

For simplicity we assume that $\{l_t\}$ is *i.i.d.* Thus we have

$$Var(I_t) = \frac{c_{12}^2 Var(l_t)}{\left(1-c_{12}^2\right)\left[(1+r)(1-\zeta)\right]^2} w^2.$$

The impact of $\zeta$ on $c_{12}$ in the individual wealth accumulation equation (25) represents the inheritance effect of the estate tax on the stationary wealth distribution. The higher the estate tax $\zeta$ the lower $c_{12}$.[27] Thus the inheritance effect of the estate tax increases the long-run wealth inequality. The impact of $\zeta$ on the lump-sum transfer $g$ in the individual wealth accumulation equation (25) represents the redistribution effect of the estate tax.

[27] Note that

$$c_{12} = \frac{1}{\left[(1+r)(1-\zeta)\right]^{-1} + \left[(1+r)(1-\zeta)\right]^{-\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}}.$$

We can calculate the coefficient of variation,

$$
\begin{aligned}
& CV(I_t) \\
= \; & \frac{\sqrt{Var(I_t)}}{\bar{I}} \\
= \; & \frac{1-(1+r)\left(1-[(1+r)(1-\zeta)]^{-\frac{1}{\eta}}\chi^{-\frac{1}{\eta}}\right)}{(1+r)(1-\zeta)} \frac{c_{12}}{\sqrt{1-c_{12}^2}} \sqrt{Var(l_t)}.
\end{aligned}
$$

To illustrate the impact of the estate tax $\zeta$ on the lump-sum transfer $g$ and that of the estate tax $\zeta$ on the long-run wealth inequality, we implement a simple calibration exercise. We pick $\eta = 2$, $\chi = 0.8$, $r = 2$, and $w = 1$. We assume that one generation lasts for 30 years. Thus $r = 1$ corresponds to the annual interest rate of 3.7%. We increase the estate tax $\zeta$ from 0.1 to 0.5. Figure A.1 shows that the higher the estate tax $\zeta$ the lower $g$. Thus the redistribution effect of the estate tax increases the long-run wealth inequality.

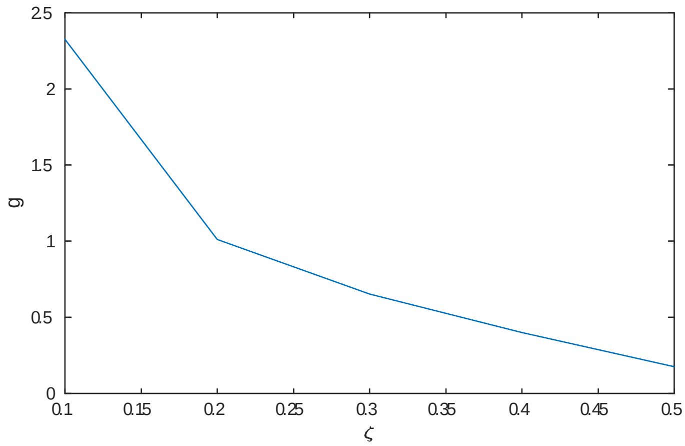



Figure A.1: The impact of the estate tax on the transfer

We also investigate the net effect of the estate tax on the long-run wealth inequality. We assume that $l_t \sim U[0,2]$. Thus $E(l_t) = 1$ and $Var(l_t) = \frac{2}{3}$.

Figure A.2 shows that the higher the estate tax $\zeta$ the higher the CV of the long-run wealth inequality.

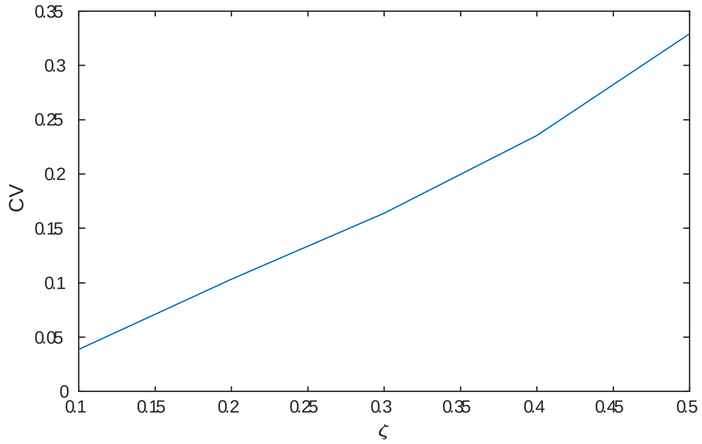



Figure A.2: The impact of the estate tax on the CV

Figure A.3 shows that the higher the estate tax $\zeta$ the higher the Gini coefficient of the long-run wealth inequality. Figures A.2 and A.3 show that the estate tax increases the long-run wealth inequality. This result is reasonable since both the inheritance effect and the redistribution effect of the estate tax increase the long-run wealth inequality.

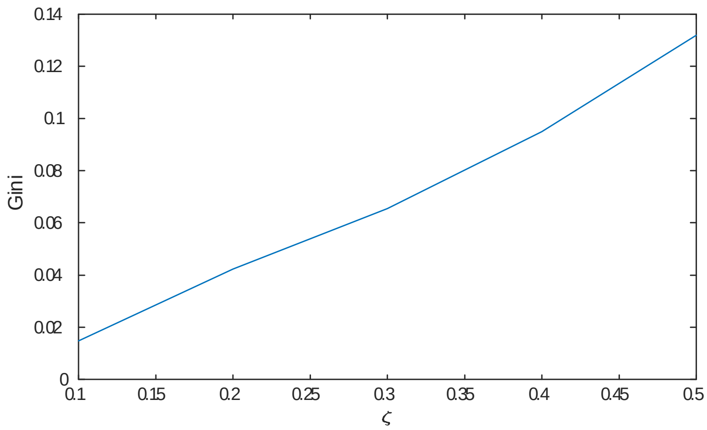



Figure A.3: The impact of the estate tax on the Gini coefficient