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Observation equivalence holds in the hyperbolic discounting models such as Laibson (1996), Barro (1999), and Krusell et al (2002). We study a hyperbolic discounting model where the policy function cannot be replicated by a geometric discounting model. Under the logarithmic utility and Cobb−Douglas production, we obtain the explicit solution for consumer's consumption−saving decision. Different from the literatures of exponential discounting, our model shows that the habit persistence affects consumer's consumption−saving decision. Therefore, observational equivalence does not hold in our hyperbolic discounting model.

We would like to thank Professor Christian Gollier, Professor Heng−fu Zou, and Dr. Wang Yaping for their helpful comments. Of course, we are responsible for any remaining errors. This research was sponsored by the program for New Century Excellent Talents in University and the National Natural Science Foundation of China under project 70532002.
Citation: Liutang, Gong and Zhu Shenghao, (2006) "Does observational equivalence always hold in hyperbolic discounting models?." *Economics Bulletin,* Vol. 5, No. 8 pp. 1−8
Submitted: April 4, 2006. **Accepted:** May 17, 2006.
URL: http://economicsbulletin.vanderbilt.edu/2006/volume5/EB−06E20003A.pdf

Does observational equivalence always hold in hyperbolic discounting models?[1]

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Abstract

Observation equivalence holds in the hyperbolic discounting models such as Laibson (1996), Barro (1999), and Krusell *et al* (2002). We study a hyperbolic discounting model where the policy function cannot be replicated by a geometric discounting model. Under the logarithmic utility and Cobb-Douglas production, we obtain the explicit solution for consumer's consumption-saving decision. Different from the literatures of exponential discounting, our model shows that the habit persistence affects consumer's consumption-saving decision. Therefore, observational equivalence does not hold in our hyperbolic discounting model.

Key Words: Observational equivalence; Habit persistence; Hyperbolic discounting.

JEL Classification: D11, E21.

1 Introduction

Since Strotz (1956), the hyperbolic discounting has become an important tool to discuss the time-inconsistent decision-makings in economics, especially in the theory of consumer's consumption decision. For example, Laibson (1996), Barro (1999), and Krusell *et al* (2002) have studied consumer's consumption-saving decision in various hyperbolic discounting models. However, the observational equivalence, which means that the consumption-saving decision in a hyperbolic discounting model is equivalent to those in a model with some standard exponential discounting rate, has been proved true in these literatures. Thus, we cannot distinguish the hyperbolic discounting from standard exponential discounting with the aid of these literatures. Therefore, whether we can find a model to distinguish the hyperbolic discounting from the exponential discounting needs to be studied further. This paper tries to fill the gap to set up a model to distinguish the hyperbolic discounting from standard exponential discounting.

This paper introduces the habit persistence into the hyperbolic discounting model. With the help of the effect of habit persistence on consumer's consumption-saving decision, we can obtain a policy function, which cannot be replicated by a

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standard geometric discounting model. For the specified utility function and production function, the explicit solution for consumption-saving decision has been derived. Different from the standard exponential discounting, it is found that the habit persistence affects the consumer's policy function in this paper, which can be used to distinguish the hyperbolic discounting from the standard exponential discounting.

The rest of this paper is organized as follows: We set up the basic hyperbolic discounting model with the consideration of habit persistence in section 2. The explicit solution for consumer's consumption-saving decision has been derived in this section. We conclude our paper in section 3.

2 The Model

2.1 The basic set up

We introduce the habit persistence into Krusell et al (2002) decentralized economy model. The agent has a transitional habit formation. The agent' instantaneous utility at period t is specified as

$$u(c_t, c_{t-1}) = \log c_t + \gamma \log c_{t-1},$$

where γ represents the extent to which the habit persistence influences the current period utility, or the extent to which the agent's memory of consumption in the last period influences the current period utility. In order to exclude the trivial result, we impose:

$$\gamma < \min\{-\frac{1}{\delta}, -\frac{1}{\delta\beta}\} \quad \text{or} \quad \gamma > \max\{-\frac{1}{\delta}, -\frac{1}{\delta\beta}\}.$$

where $0 < \delta < 1$ and $\beta > 0$ are positive constants, they represent the agent's time discounted rate.

The individual's capital stock is k_t and the aggregate capital stock of the economy is $\overline{k}_t$. All the markets are perfectly competitive. The individual agent is a price-taker. The aggregate capital stock $\overline{k}_t$ is given as a constant for the individual. Thus, the budget constraint for the agent can be written as:

$$c_t + k_{t+1} = r(\overline{k}_t)k_t + w(\overline{k}_t), \tag{1}$$

where $r(\overline{k}_t)$ and $w(\overline{k}_t)$ are the interest rate and the wage rate, respectively.

Under the quasi-hyperbolic discounting, the agent's discounted utility at time t can be written as

$$U_t = u(c_t, c_{t-1}) + \beta \sum_{i=1}^{+\infty} \delta^i u(c_{t+i}, c_{t+i-1}), \tag{2}$$

where $u(c_t, c_{t-1})$ is the instantaneous utility function of the agent; $0 < \delta < 1$ and $\beta > 0$ are positive constants, they represent the agent's time discounted rate. When $\beta = 1$, the agent has standard geometric preferences; When $0 < \beta < 1$, the agent has excessive short-run impatience; When $\beta > 1$, the agent has excessive short-run patience.

Under the budget constraint (1), the agent chooses the consumption-saving decision to maximize the discounted utility in equation (2). Output is produced by the neoclassical production function $f(\overline{k}_t)$. In the perfectly competitive market, the firm's profit maximization presents that

$$r_t = f'(\overline{k}_t),\ w_t = f(\overline{k}_t) - f'(\overline{k}_t)\overline{k}_t, \tag{3}$$

which mean that the marginal productivity of capital equals the interest rate, r_t, and the marginal productivity of labor equals the wage, w_t. In order to obtain the explicit solution, we specify the production function as Cobb-Douglas form

$$f(\overline{k}_t) = A\overline{k}_t^{\alpha},$$

where A and $0 < \alpha < 1$ are constants. The capital is fully depreciated.

2.2 Recursive competitive equilibrium and explicit solutions

In this subsection, we derive the explicit solution for the recursive competitive equilibrium. Similar to Krusell *et al* (2002), our model suppose that the agent makes his decision taking as given the prices as functions of the aggregate state $\overline{k}$, $r(\overline{k})$ and $w(\overline{k})$, and a law of motion for aggregate capital, $\overline{k}' = G(\overline{c}_{-1}, \overline{k})$, and the other individuals' decision rule of saving; $k' = g(k, c_{-1}, \overline{k})$. The recursive equilibrium requires three state variables for the individual: the individual current capital, k, the individual's last period consumption, c_{-1}, and the aggregate current capital, $\overline{k}$. The middle one represents the habit while the last one reflects prices. The current self solves the following problem at time t

$$V_0(k, c_{-1}, \overline{k}) = \max_{c,k'}\{u(r(\overline{k})k + w(\overline{k}) - k') + \beta\delta V(k', c, \overline{k}')\}, \tag{4}$$

where $V(k, c_{-1}, \overline{k})$ satisfies

$$V(k, c_{-1}, \overline{k}) = u(r(\overline{k})k + w(\overline{k}) - g(k, c_{-1}, \overline{k})) + \delta V(g(k, c_{-1}, \overline{k}), c, \overline{k}'). \tag{5}$$

The solution to optimization problem (4) is given by $\widetilde{g}(k, c_{-1}, \overline{k})$, and the corresponding law of motion for aggregate capital is $\widetilde{G}(\overline{c}_{-1}, \overline{k})$. We have a solution to the agent's game (i.e., to the game between the different selves) if the fixed-point conditions $\widetilde{g}(k, c_{-1}, \overline{k}) = g(k, c_{-1}, \overline{k})$ and $G(\overline{c}_{-1}, \overline{k}) = \widetilde{G}(\overline{c}_{-1}, \overline{k})$ are satisfied for all k, c_{-1}, $\overline{c}_{-1}$and $\overline{k}$. Therefore,

Definition 1: *A recursive competitive equilibrium for the economic system consists of a decision rule, $g(k, c_{-1}, \overline{k})$, a value function, $V(k, c_{-1}, \overline{k})$, pricing functions $r(\overline{k})$ and $w(\overline{k})$, and a law of motion for aggregate capital, $\overline{k}' = G(\overline{c}_{-1}, \overline{k})$, such that*

1. Given $V(k, c_{-1}, \overline{k})$, $g(k, c_{-1}, \overline{k})$ solves the maximization problem (4) and $V(k, c_{-1}, \overline{k})$ satisfies the functional equation (5);

2. $r(\overline{k})$ and $w(\overline{k})$ maximize the firm's profit, i.e., $r(\overline{k}) = f'(\overline{k})$ and $w(\overline{k}) = f(\overline{k}) - \overline{k}f'(\overline{k})$;

3. The law of motion for aggregate capital derived from the current self's decision is consistent with the law of motion for aggregate capital; i.e., $g(\overline{k}, \overline{c}_{-1}, \overline{k}) = G(\overline{c}_{-1}, \overline{k})$.

Solving problems (4) and (5), we arrive

Proposition 1. *The recursive competitive equilibrium is given by:*

1. $V(k, c_{-1}, \overline{k}) = a + b\log\overline{k} + e\log(k + \varphi\overline{k}) + d\log c_{-1}$, where $d = \gamma$, $e = \frac{1+\delta\gamma}{1-\delta}$, $b = \frac{(1+\delta\gamma)(\alpha-1)}{(1-\delta)(1-\alpha\delta)}$, and $\varphi = \frac{(1-\alpha)[(1+\delta\beta\gamma)(1-\delta)+\delta\beta(1+\delta\gamma)]}{\alpha(1+\delta\beta\gamma)(1-\delta)}$;

2. $g(k, c_{-1}, \overline{k}) = \frac{\delta\beta(1+\delta\gamma)}{1-\delta+\delta\beta+\delta\beta\gamma} r(\overline{k})k$; and

3. $G(\overline{c}_{-1}, \overline{k}) = g(\overline{k}, \overline{c}_{-1}, \overline{k}) = \frac{\delta\beta(1+\delta\gamma)}{1-\delta+\delta\beta+\delta\beta\gamma}\alpha A\overline{k}^{\alpha}$.

Proof. See the Appendix A.

From proposition 1, we can easily derive the explicit solutions for consumption and investment decisions:

$$c_t = \frac{(1-\delta)(1+\delta\beta\gamma)}{1-\delta+\delta\beta+\delta\beta\gamma}\alpha A\overline{k}_t^{\alpha-1}k_t + (1-\alpha)A\overline{k}_t^{\alpha}, \quad (6)$$

$$k_{t+1} = \frac{\delta\beta(1+\delta\gamma)}{1-\delta+\delta\beta+\delta\beta\gamma}\alpha A\overline{k}_t^{\alpha-1}k_t. \quad (7)$$

From equation (7), the explicit solution for the aggregate capital accumulation can be derived as:

$$\overline{k}_{t+1} = \frac{\delta\beta(1+\delta\gamma)}{1-\delta+\delta\beta+\delta\beta\gamma}\alpha A\overline{k}_t^{\alpha}. \quad (8)$$

The standard exponential discounting model becomes a special case of our model. Setting $\beta = 1$ in equations (6) and (7), we have

$$c_t = (1-\delta)\alpha A\overline{k}_t^{\alpha-1}k_t + (1-\alpha)A\overline{k}_t^{\alpha}, \quad (9)$$

$$k_{t+1} = \delta\alpha A\overline{k}_t^{\alpha-1}k_t. \quad (10)$$

From equations (9) and (10), we know that the individual's decisions are independent of the habit persistence parameter γ, i.e., the habit persistence does not affect the individual's decisions in the standard exponential discounting model. However, from equations (6) and (7), we find that the habit persistence affects the consumer's consumption-saving decision. Therefore, if we can observe the habit persistence parameter γ, the observational equivalence between the hyperbolic discounting and exponential discounting would not hold yet. This is a critical point that we can be used to distinguish the hyperbolic discounting from the standard exponential discounting.

This result cannot be obtained in a hyperbolic discounting model without habit persistence. Observational equivalence always holds in that case. When $\gamma = 0$, equations (6) and (7) are reduced to

$$c_t = (1 - \frac{\delta\beta}{1-\delta+\delta\beta})\alpha A\overline{k}_t^{\alpha-1} k_t + (1-\alpha)A\overline{k}_t^{\alpha}, \quad (11)$$

$$k_{t+1} = \frac{\delta\beta}{1-\delta+\delta\beta}\alpha A\overline{k}_t^{\alpha-1} k_t, \quad (12)$$

which are the results presented by Krusell *et al* (2002). A model under the standard exponential discounting with discount rate δ' can replicate the policy function of the model with the hyperbolic discounting, where

$$\delta' = \frac{\delta\beta}{1-\delta+\delta\beta}.$$

This is exactly the observational equivalence between the hyperbolic discounting model and the standard discounting model. This equivalence result holds for both aggregate and individual variables.

2.3 Implication for the empirical test for hyperbolic discounting

From the discussion above, we know that the habit persistence will affect consumer's consumption-saving decision in the model with hyperbolic discounting, while it does not affect the consumer's decision in the model with standard discounting. Therefore, we can use it to set up an empirical test to distinguish the hyperbolic discounting from the standard exponential discounting. Similar to Garcia, Renault, and Semenov (2002), we can test the saving rate of different countries, controlling other factors and varying the habit persistence factor, to find the evidence of the existence of the hyperbolic discounting. If the hypothesis of the existence of hyperbolic discounting is true, we should find that people who have different habit persistence parameter γ have different saving rates. On the other hand, if people with different habit persistence have the same saving rate, the hypothesis of the existence of hyperbolic discounting is rejected and the opposite hypothesis of the existence of the exponential discounting could not be rejected. This will leave for further research.

3 Conclusion

In this paper, we introduce habit persistence into Laibson (1996) and Krusell *et al* (2002)'s model to examine the effects of hyperbolic discounting and habit persistence on consumption and investment decisions. For the specified utility function and production function, we derive the explicit solutions for the consumption and investment decisions and find that both habit persistence and hyperbolic discounting affect consumer's consumption-saving decision. The marginal propensity to consumption depends on the parameters of habit persistence and hyperbolic discounting. Therefore, if the habit persistence is observed, the observational equivalence in Laibson (1996), Barro (1999), and Krusell *et al* (2002) will not hold yet in this paper. This helps us to set up empirical methods to distinguish the hyperbolic discounting from the standard exponential discounting.

Further researches should set up econometric models to implement this theoretical method and test the effects of hyperbolic discounting on economy.

4 Appendix A: Proof of Proposition 1

In order to prove the proposition 1, we postulate the forms for the value function and the law of motion for aggregate capital as

$$V(k, c_{-1}, \overline{k}) = a + b \log \overline{k} + e \log(k + \varphi \overline{k}) + d \log c_{-1},$$
$$G(\overline{c}_{-1}, \overline{k}) = sA\overline{k}^{\alpha},$$

where a, b, e, and d are to be determined.

Solving the current self's problem, we obtain

$$g(k, c_{-1}, \overline{k}) = \frac{\delta\beta e\alpha A\overline{k}^{\alpha-1}k + [\delta\beta e(1-\alpha) - (1+\delta\beta d)s\varphi]A\overline{k}^{\alpha}}{1 + \delta\beta e + \delta\beta d}.$$

Subsitituting this decision rule into equation (5), we have $d = \gamma$, $e = \frac{1+\delta\gamma}{1-\delta}$, $b = \frac{(1+\delta\gamma)(\alpha-1)}{(1-\delta)(1-\alpha\delta)}$, $\varphi = \frac{1-\alpha}{\alpha-s}$.

Substituting $\varphi = \frac{1-\alpha}{\alpha-s}$ into the individual's decision rules and setting $g(\overline{k}, \overline{c}_{-1}, \overline{k}) = G(\overline{c}_{-1}, \overline{k})$, we obtain $s = \frac{\delta\beta(1+\delta\gamma)\alpha}{1-\delta+\delta\beta+\delta\beta\gamma}$ and $\varphi = \frac{\delta\beta e-(1+\delta\beta d)s\varphi}{1+\delta\beta e+\delta\beta d}$. Therefore, we have the explicit solution for $g(k, c_{-1}, \overline{k})$ and $G(\overline{c}_{-1}, \overline{k})$.

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